1. (From “Introduction to Algorithms”) A sequence of $n$ operations is performed on a data structure. The $i$th operation costs $i$ if $i$ is an exact power of 2, and 1 otherwise. Determine the amortized cost per operation in two ways: (i) using the accounting method; (ii) using the potential method.

2. (From “Introduction to Algorithms”) Design a data structure to support the following two operations for a set $S$ of integers:
   - Insert($S$, $x$) inserts $x$ into $S$.
   - Delete-Larger-Half($S$) deletes the largest $\lceil S/2 \rceil$ elements from $S$.

   Explain how to implement this data structure so that any sequence of $m$ operations runs in $O(m)$ time.

3. (From “Computational Geometry” by de Berg, van Kreveld, Overmars, and Schwarzkopf) Let $A$ be a set of numbers. Analyze the expected running time of the following procedure RandMax($A$). The set $A$ in the first call to RandMax contains $n$ distinct numbers.

   RandMax($A$)
   (a) If $|A| = 1$, return the number in $A$.
   (b) Otherwise, choose a random number $x \in A$.
      i. $y := \text{RandMax}(A \setminus \{x\})$.
      ii. If $y \geq x$, return $y$.
      iii. Otherwise, compare $x$ with all other elements in $A$ to confirm that $x$ is larger than them. Return $x$. 
