A Model of Sequential Reforms and Economic Convergence*

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Abstract

A simple growth model is developed to explain how economic convergence triggers and is also sustained by the endogenous and successive institutional reforms. International learning externality facilitates economic convergence until a growth bottleneck is reached, at which point convergence stops unless the institution is improved. After the reform, convergence resumes until a new bottleneck is encountered, which triggers another reform, so on and so forth. Recursive method is used to characterize this non-stationary dynamic problem. Three analytical results are obtained. First, reforms and hence convergence cannot last forever (there must be a finite number of institutional adjustments), and each reform occurs precisely when the new growth bottleneck just becomes binding. Second, the magnitude of the reform (adjustment size) is strictly monotonic or constant over time when the fixed adjustment cost is negligible. Third, an incessant economic convergence is achieved as a result of a sequential and cumulative underlying institutional buildings until the economy reaches the balanced growth path, on which a permanent GDP gap may persist between the two countries. Moreover, when the reform size is strictly increasing over time, the long-run GDP gap is bigger than that when the reform size is diminishing over time. Our analysis suggests that a politically more powerful government should adopt more gradual reforms than a weak government.

Key Words: Economic Convergence, Growth Models, Growth Bottlenecks, Optimal Reform

JEL Classification Codes: O11,O21,O43

1 Introduction

Most economists today would agree that institutions and policies are fundamental for the long-run economic growth (see North, 1990, Hall and Jones, 1999, Acemoglu, Johnson, and Robinson, 2005, *etc.*).[1] It is natural to ask how to improve the inefficient institutions in the developing countries in order to help them achieve economic convergence to the rich countries. The last several decades did witness institutional transitions in China, India, Russia, Vietnam, and many other countries. Some were successful but others were not. To understand why, it is crucial to not only figure out the optimal schemes of institutional adjustment but also fully understand the interactive macroeconomic mechanics between economic reforms and the associated growth dynamics.

After summarizing a total of eighty-three episodes of economic accelerations worldwide in the twentieth century, together with many other concrete development anecdotes, Rodrik (2005) concludes that accelerations typically occurred when some institutional bottleneck was relaxed. He argues that to ignite economic convergence in a developing economy may only require a one-time small change in some institution or policy, but to sustain the convergence it would need a process of *cumulative institutional building* along the way:

"*In the long run, the main thing that ensures convergence with the living standards of advanced countries is the acquisition of high-quality institutions. The growth-spurring strategies have to be complemented over time with a cumulative process of institution building to ensure that growth does not run out of stream....*"

Unfortunately, however, so far there exist few, if any, theoretical models that explicitly characterize how the convergence occurs with an endogenous "cumulative process of institutional building" in the standard growth framework.[2] In this paper I aim to help fill this gap. My modelling is disciplined to minimize the deviation from the standard growth framework. The focus is on the analytical characterization and economic description of both the general economic logic and the implied growth dynamics of the "cumulative and sequential reforms". No attempt is made to propose a brand-new theory about how to identify some novel institutional constraints or to provide a novel solution to reform certain specific inefficient institutions or policies.

To help theorize this general process, it may be useful to start with some very specific and concrete examples. Consider the development process of the potato cluster in a county called Anding in Northwest China. Before the Household Responsibility System (HRS) was adopted in the late 1970s, all the farmers worked for the commune and were equally paid regardless of individual effort. Naturally the output was low and people could not even feed themselves. After the agricultural reform, HRS was implemented, which

[1] Glaeser et al (2004) cast doubt on many empirical researches that support this viewpoint.

[2] Acemoglu, Aghion and Zilibotti (2006) show that, in order to achieve sustained development, the growth mode should switch from the investment-based strategy to innovation-based strategy when a country gets closer to the world technology frontier. However, they do not emphasize the "cumulative process" of sequentially relaxing the new institutional contraints.

allowed individual households to claim their residuals after fulfilling some fixed quota. This reform solved the incentive problem, which was the main institutional bottleneck for growth at that stage, thus output grew rapidly afterwards. However, the next growth hurdle appeared soon: the land was so infertile that it was difficult to raise the output of traditional crops such as wheat or sorghum. To solve this problem, the local farmers and the local government invited some agricultural experts from Beijing to seek solutions and suggestions. It was soon discovered that some types of potatoes were magically suitable for the land conditions of that region. Those potatoes were introduced into the Anding County, together with the information about how to grow them efficiently. Thanks to this technology adoption, the aggregate agricultural output increased tremendously. The farmers could not only feed themselves sufficiently but also have surplus potatoes to sell. Now came the new growth bottleneck: local farmers had no information about the potato prices in the major national potato market such as the one in the city of Zhengzhou, which is far from Anding County. Consequently, farmers could only sell their surplus potatoes at low prices to a handful of local intermediaries with monopoly power. To get rid of this bottleneck of income growth, some farmers jointly hired a person to physically stay in Zhengzhou collecting and reporting the price information, and later the county government of Anding established a new office located in Zhengzhou helping disseminate the potato price information freely to all the farmers in Anding. As information hurdle was dismantled, local farmers got good bargains for their surplus potatoes and therefore the output skyrocketed. However, the fast output growth soon made the transportation capacity become the new growth bottleneck, because long-way transportation had to rely on the trains, which unfortunately had only two carts allocated to that county. After some personal contact and negotiations between the Anding government and China's Ministry of Railways, four more carts were added to the trains and the transportation constraint was effectively relaxed and the output continued to climb. The neighboring counties now also started to mimic the practice of Anding country after witnessing the success of its potato industry. So even six carts were not enough. Then local people cooperated with the local government and found an innovative way to hoard potatoes during the harvest season and also tried to produce and export the higher value-added processed products of potatoes rather than raw potatoes, which naturally led to further growth in the local GDP of Anding county (see Zhang and Hu, 2011).

From this process, we can see that the industrial development encounters distinctive bottlenecks at different stages, and further growth becomes admissible only when the new binding constraints are dismantled in a timely fashion. Of course, this is only an example about a particular industry in a county, and for the local industries the growth bottlenecks are often more about the provision of relevant public goods and services (or technological innovation) instead of the change in the fundamental institutions such as the property rights, financial institutions, law enforcement, or constitutions, *etc.* Nevertheless, similar logic also carries to the analysis of the aggregate macroeconomic performance in a developing country which undertakes sequential reforms on various inefficient institutions or policies that may become binding constraints for growth over the course of economic development. Optimal reforms at the country level do not have to be cross-the-board change either.[3] The key task I set in this paper is precisely to provide a theoretical model that is able to describe this kind of interactive process between economic growth/convergence and sequential relaxations of the newly-arising constraints by cumulative institutional buildings.

The institutional bottlenecks are typically different in nature for different countries at different development stages and for our purpose, we need to incorporate all the different growth bottlenecks and reforms into a unified framework. To maintain analytical tractability and highlight the general features in the interactive mechanics of institutional change and convergence, I have to abstract away all the specific institutional details and political economy considerations. Instead, I address the following normative question: what would be the first-best scheme of institutional adjustments and the associated growth performance? Certainly, no question can be simpler than this and the answer cannot be expected to provide an immediate policy prescription for how to carry out specific reforms in the reality. Nevertheless, from a theoretical point of view, it seems important to have a confident answer to this basic academic question before any deeper and more complicated institutional analysis can be made.

Following Lucas (2009), I will study this question in the context of an open economy and ask how to reduce a developing country's existing barriers to absorbing foreign better technology or the learning externality in the human capital.[4] To keep our analysis simple, I will model the relevant "institutions" or "policies", a multi-dimension complicated object, just as one single abstract "institutional barrier" variable that hampers the inflow of production-related knowledge from more developed economies, either through human capital externality and international spillover (see Klenow and Rodriguez-Clare (2005) for an excellent survey), or through the more purposeful technology adoption and imitation (See, for example, Eaton and Kortum, 1999; Parente and Prescott, 2000).[5] Economic reform in this model is therefore nothing but the reduction of this institutional barrier variable.

Following the pertinent growth literature, I assume that a less developed economy grows as its human capital accumulates thanks to the learning externality (spillover) from a representative developed economy. So the initial large gap in GDP between the two countries enables the poor one to catch up. The convergence stops when their gap shrinks to a threshold value, which depends on the institutional barrier variable. This bottleneck for convergence is referred as the learning externality constraint being

[3] Cander (2006), Easterly (2002), Rodrik (2005) and World Bank (2008) all provide many real-life cross-country examples how a mild policy or institutional change in certain industry suddenly activated that industry, which became increasingly successful and ultimately kicked start the economic accelerations of the whole country. In particular, Rodrik (2010) advocates the approach of growth diagnostics and shows that the binding institutional constraints are generally different across different countries. For example, he shows that the financial constraints are the key hurdle for some countries, but the lack of investment opportunities are more crucial in others.

[4] See more discussions on the convergence in the open economy, please refer to Sachs and Warner (1995) and Ventura (2005), just to name a few..

[5] Stokey (2009) examines explicitly the different roles played by human capital and technology diffusion in the catching up process.

binding. The Ramsey planner formulates a dynamic adjustment scheme which specifies when and how much to change this barrier variable in order to sustain the human capital spillover and hence the catching up process. I analytically characterize how the economic growth triggers institutional changes and how the institutional change feeds back on the patterns of factor accumulation and growth dynamics. The main theoretical results are the following. First, reforms and hence convergence cannot last forever (there must be a finite number of institutional adjustments), and each reform occurs precisely when the learning constraint just becomes binding. Second, the magnitude of the reform (adjustment size) is strictly monotonic or constant over time when the fixed adjustment cost is negligible. Third, the optimal sequential and accumulative institutional building allows the economy to catch up incessantly at the beginning but a permanent GDP gap may persist between the two countries on the balanced growth path, as reform stops when the potential gain is outweighed by the adjustment cost. Moreover, when the reform size is strictly increasing over time, the long-run GDP gap between the two countries is bigger than that when the reform size is diminishing over time. I place more emphases on the level effect than the growth effect.[6]

This model highlights the importance of "cumulative and sequential" institutional building that underlies the economic convergence process. In the first-best equilibrium, the course of convergence appears to be fully dictated by the human capital accumulation and it is not affected by any explicit institutional variables. However, the model clearly points out that, beneath the apparent GDP dynamics, the key force that really ignites and sustains the economic convergence is actually the accumulative institutional building process, which has not been emphasized sufficiently in the existing neoclassical and endogenous growth models. Moreover, if we may interpret a binding learning constraint as an "crisis" (because this constraint strangles further convergence), then the optimal timing of reforms predicted by the model is quite consistent with the empirically verified "crisis hypothesis" in the reform literature, which states that the reform is more likely to occur after a "crisis" appears.[7]

Depending on how to interpret the fixed cost and the variable cost associated with institutional building, the model also sheds light on the optimal reform strategy. One implication is that the optimal reform scheme may depend on whether the government is powerful or not, as it determines the relative importance of the fixed adjustment cost versus the variable adjustment cost. A strong government, which may be authoritarian, tends to formulate and implement a reform plan more quickly than a weak government (especially an unconsolidated democracy), which may find itself unable to initiate or push forward a reform unless all the special interest groups have reached certain agreement beforehand. Thus the stronger the government, the smaller the fixed cost for each reform, *ceteris paribus*. The opposite is true for the variable adjustment cost as the weak

[6] A standard result in the growth models without institutional change is that the developing and the developed countries will grow at the same speed on the balanced growth path, as can be clearly seen from the law of motion in the human capital diffusion. But the level difference could be still enormous. See more argument in Parente and Prescott (2000).

[7] See for example, Alesina, Ardagna, and Trebbi, 2006; Drazen and Grilli, 1993, *etc.*.

government has already tried minimizing the possible negative impact for each group.[8] This asymmetry, according to the model, will imply that the economy with a stronger government should have a more gradual economic reform while one with a weak government should have a more radical reform (less frequent but in a larger magnitude each time) to avoid the huge fixed adjustment cost. These results are in sharp contrast to the standard "Washington Consensus", which stipulates that all the institutions must be reformed simultaneously and the faster the better, regardless of the cost structures for reforms that depend on the political institutions.[9]

North (1990), among others, categorizes institutions into formal institutions (such as legal rules and democratic political systems) and informal institutions (such as moral codes and common practice). Dixit (2004) argues that setting up formal institutions requires high fixed costs but low marginal (variable) costs, whereas the opposite is true for informal institutions. With this interpretation, my model implies that the optimal reform tends to be quicker when many informal institutions are still on the reform list, especially at the early stage of reform, but the reform gets slower when formal institutions need to be changed, presumably at the later stage of reform.

In the growth literature, economic convergence is usually examined with institutional barriers taken as exogenous and/or fixed.[10] Numerous studies have already shown how economic growth is hampered by poor protection of property rights, corruptions, or bad policies.[11] In addition, various political-economy models are constructed to study how foreign better technologies are sometimes blocked by the domestic vested-interest clique.[12] Unfortunately, however, none of these studies have explicitly addressed the reverse questions: how to reform the bad institution dynamically to achieve economic betterment and how does the equilibrium growth path interact with the accumulative reforms? There certainly exists a vast but somewhat orthogonal literature on institution transitions and reforms.[13] Unfortunately, most of these models are not formulated within the standard growth framework, and therefore do not fully characterize the interactive macroeconomic "mechanics" between the factor accumulation and economic convergence. Nor is the feature of sequentially relaxing growth bottlenecks emphasized enough. This paper tries to bridge the gap between these two different strands of literature.

[8] There is a recently growing literature that emphasizes the importance of state capacity and the drawback of a weak government in economic development, see Blanchard and Shleifer (2001), Acemoglu (2005), Besley and Persson (2009).

[9] The "Washington Consensus" has received wide criticisms and enormous challenges, as it seems incapable of explaining many empirical facts including, for example, the stagnation of the average developing countries in 1980-1998 although they all moved toward the "Washington Consensus" (Easterly, 2001). The skepticism reached its peak after the former Soviet Union and some other East European countries experienced unexpectedly huge economic difficulties after following the recipe of "Washington Consensus" such as adopting the shock therapy, in sharp contrast with the unexpected success of China, which has instead adopted a more pragmatic and piece-meal reform strategy (see Stiglitz, 1998; Rodrik, 2005; World Bank, 2008; Lin 2009, etc.).

[10] See, Barro and Sala-I-Martin, 1992; Lucas, 2002, 2009; Ngai, 2004; Stokey, 2009, *etc.*

[11] A nice survey is provided by Acemoglu, Johnson and Robinson, 2005.

[12] See, for example, Krusell and Rios-Rull, 1996; Parente and Prescott, 2000, etc.

[13] See, among others, Bruno1972; Murphy, Shleifer and Vishny, 1992; Dewatripont and Roland, 1992; Lau, Qian, and Roland, 2000; Roland, 2002; Caselli and Gennaioli, 2008.

The rest of the paper is organized as follows: Section 2 describes the model economy. In Section 3, I characterize the optimal institutional adjustment and the associated macro dynamics. Section 4 discusses some possible avenues for future research. Section 5 concludes.

2 Model Environment

Consider a continuous-time deterministic world. There is a developing economy populated by a unit mass of identical households. I will focus on the artificial social planner's optimal institutional adjustment.[14] The Ramsey government wishes to maximize a representative household's total discounted utility:

$$\int_0^\infty c(t)e^{-\rho t}dt, \tag{1}$$

where ρ is the discount rate. The assumption of infinite inter-temporal elasticity of substitution can help us focus on the institutional change problem by tremendously simplifying the consumption analysis.[15] Following Lucas (2009), I assume that a representative household is endowed with one unit of labor, which is inelastically supplied to produce one homogeneous good with the following simple technology:

$$f(h) = h,$$

that is, one unit of human capital, h, combined with one unit of labor, can produce one unit of storable good, which can be either consumed or used to pay the cost of institution adjustment.[16] The initial human capital is h_0. Thus GDP dynamics is equivalent to the change of human capital stock.

There is also a developed country with the same production technology. $H(t)$ denotes its human capital stock at time t, which grows exogenously at a constant exponential

[14] The model is cast as a central planner problem rather than a competitive equilibrium problem for reasons beyond the second welfare theorem and modeling convenience: (1) some important markets may be missing in the less developed economy, hence resource allocation may not fully operate through the market mechanism, and (2) in reality the central governments in many transitional economies have a far greater administrative power than their counterparts in the developed economies, both in terms of shaping and changing the institutions. In fact, in many developing countries such as China or India, the central governments do have and implement very formal and extensive five-year, ten-year or twenty-year plans to reform the economic institutions.

[15] If we choose a non-degenerate CRRA utility function, the growth rate will certainly depend on the intertemporal elasticity of susbtitution. However, that would tremendously complicate the analysis of institutional adjustment without generating any useful new insights. Recall this paper wants to emphasize the level effect instead of the growth effect.

[16] Here human capital actually might refer to the combination of all the intangible accumulative production factors including technology. We do not differentiate the learning externality in human capital and technology spillover. Endogenous decisions on the accumulation of human capital could have been introduced in the standard textbook ways, but this complication does not add sufficient new insights for our current purpose.

speed $g_H \geq 0$. $H(0)$ is normalized to unity. Let $x(t) \equiv \frac{h(t)}{H(t)}$ measure the two countries' income gap at time t. So $x(0) = h_0$. Due to the positive learning externality in human capital, $h(t)$ increases up to a constraint determined by an institutional barrier variable, $\delta(t)$, which captures all the factors that influence how effectively the economy can benefit from the externality of outside human capital at time t. For example, $\delta(t)$ may refer to the policies impeding openness such as the trade barriers, FDI policies, or intellectual property rights protection, *etc.* The more closed the economy, the larger $\delta(t)$, *ceteris paribus*. Let δ_0 denote the initial barrier value. The law of motion of the GDP gap is given by

$$\frac{dx(t)/dt}{x(t)} = \begin{cases} \mu, & if \quad x(t) \leq \frac{\eta}{\delta(t)} \\ 0, & otherwise \end{cases}, \tag{2}$$

where μ and η are both positive parameters.[17] It means that the human capital gap between the two countries shrinks at a constant exponential speed μ until the gap hits the critical value $\frac{\eta}{\delta(t)}$, at which point convergence stops unless the institutional barrier variable $\delta(t)$ is adjusted downward. This is what we mean by "institutional improvement" or "reform". η could reflect the scale of the developing economy relative to the rest of the world. A higher η implies a longer time to enjoy the convergence for any given institution barrier and GDP gap.

Institutional adjustment cost has two components: a variable cost and a fixed cost. More precisely, when the institutional variable $\delta(t)$ is adjusted from δ to δ' in a single step, the adjustment cost is given by

$$C(\delta, \delta') = \begin{cases} A\left(\frac{\delta}{\delta'}\right)^{\phi} + B, & if \ \delta \neq \delta' \text{ and } \delta' \geq \eta \\ \infty, & if \ \delta' < \eta \\ 0, & if \ \delta = \delta' \end{cases}, \tag{3}$$

where A and B are both positive parameters. Assume $\phi > 1$, implying that the adjustment cost function is convex in the adjustment size, $\frac{\delta}{\delta'}$. No adjustment ($\delta = \delta'$) naturally incurs no cost. (3) imposes a lower bound for the admissible barrier variable, η. This is to rule out the possibility that the less developed country can overtake the developed country simply by exploiting the international human capital externality, so no leapfrogging occurs: $x(t) \leq 1, \forall t$.

The adjustment cost function depends on the structural details of the political institutions, thus (3) should be seen as the reduced form for the overall cost associated with institutional changes, such as the cost due to the conflict between different special interest groups.[18] More radical reforms are more likely to trigger social instability. This

[17] This simple functional form is adopted mainly to maxmize the tractability, but it is not difficult to generalize it to a multi-value step functions (or a continuous function in the limit) so that the convergence speed depends on the gap or the institutional barrier variable as the economy catches up . Notice that by construction the less developed economy never grows at a slower pace than the developed economy in the model, but that possibility can be easily incorporated by letting the relevant value on the right hand side of (2) to be negative.

[18] In Chapter 2 of Wang (2009), a full-fledged political economy model of technology adoption is developed, which can provide a more explicit micro-foundation for this specific adjustment cost function.

is captured by the variable adjustment cost, $A\left(\frac{\delta}{\delta'}\right)^{\phi}$. For a given size of institution adjustment, countries that implement economic reforms mainly through administrative orders and centralized planning are more likely to create larger distortions and hence incur a higher social cost , so the parameter A would be larger, as compared with the pro-market reform strategies in a more deregulated market economy. On the other hand, the fixed adjustment cost B might include all the opportunity costs of developing and passing a reform plan, which can be very large especially if a long and intensive multilateral bargaining is involved. China, for instance, spent a whole decade bargaining multilaterally before finally obtaining its membership in the World Trade Organization. The more powerful and politically consolidated the central government is, the smaller fixed cost B is.[19] Another useful discussion on the cost of institutional building is given by Dixit (2004), who argues that setting up formal institutions (such as legal rules and democratic political systems) requires high fixed costs B but low marginal (variable) costs A, whereas informal institutions (such as moral codes and common practice) are the opposite.

Upward adjustment is allowed to incorporate the possibility of a reform reversal, but the benevolent government has no incentive to do so hence the relevant adjustment must be downward. The adjustment policy functional space is

$$\Delta \equiv \{\text{real function } \delta(t) : \mathbb{R}_+ \to [\eta, \delta_0] \text{ such that } \delta(0) = \delta_0 > \eta\}.$$

Since the fixed cost B is positive, conventional reasoning implies that $\delta(t)$ must be a step function due to the discontinuity of adjustments. Equivalently, the decision maker needs to find a bounded and weakly decreasing sequence $\{\delta_i\}_{i=0}^{\infty}$ and the corresponding adjustment time sequence $\{T_i\}_{i=0}^{\infty}$ with given δ_0 and $T_0 = 0$, where δ_i and T_i stand for, respectively, the value of the institutional barrier variable right after the ith adjustment and the time of that adjustment.

See Grossman and Helpman (2001) for a more systematic treatment of endogenous policies with multiple special interest groups.

[19] Blanchard and Shliefer (2001), for example, argue that one important reason why the decentralization economic reform was successful in China but failed in Russia in the 1990s is because China was more politically centralized and hence the every step of the reform was under control by the strong central government whereas in Russia the central government was too weak to maintain orders or implement effective reforms at that time, and was even outplayed by some powerful oligopoly, thus the reforms turned chaotic.

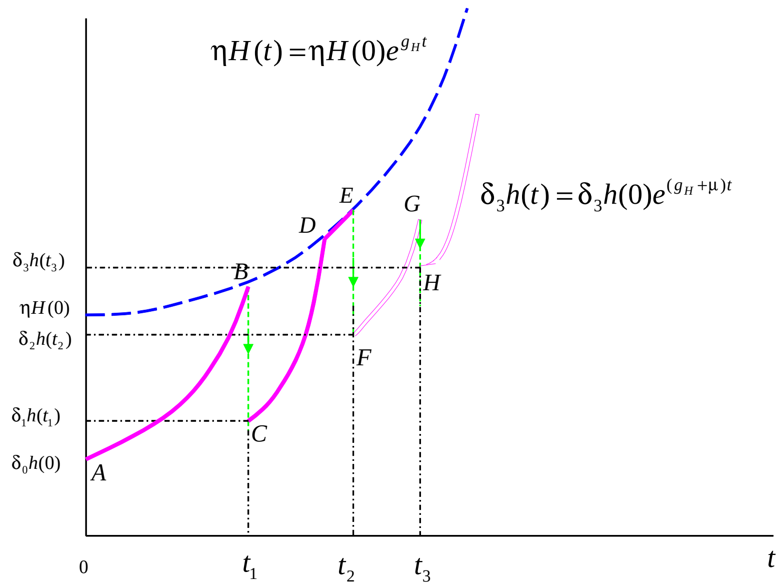

Figure 1. A Possible Adjustment Scheme

Figure 1 depicts what a possible (not necessarily optimal) adjustment path may look like. The dashed curve plots $\eta H(t)$, which grows at the exponential speed g_H. The solid curve is $h(t)\delta(t)$, which equals the less developed country's human capital stock multiplied by the barrier variable $\delta(t)$. At time 0, the less developed economy is at point A with the institution variable equal to δ_0. No institutional adjustment is made so (2) implies that $h(t)$ grows at the exponential speed $g_H + \mu$ until time point t_1, when the solid line hits the dashed line at point B. That is, the learning constraint becomes binding. The institution variable is adjusted downward from δ_0 to δ_1 at the first time of institutional adjustment t_1. After the adjustment, $\delta(t)h(t)$ jumps to point C. Note that human capital can never jump. The adjustment cost $C(\delta_0, \delta_1)$ is paid at t_1. After the institutional improvement, the learning externality constraint is relaxed so $h(t)$ continues to grow at speed $g_H + \mu$ until the learning constraint becomes binding again at point D. Since no downward institutional adjustment is made, $h(t)$ can only grow at the speed g_H afterwards and thus no catching up occurs.[20] So the solid curve overlaps with the dashed curve. The second adjustment is made at time t_2, at which the less developed economy jumps from point E to point F: the institutional variable is changed to δ_2. The economy starts to catch up again. At time t_3, the learning constraint is not binding yet, but the government may well choose to make the third adjustment at this time, so the economy jumps from point G to point H, so on and so forth. Our task is to find the optimal adjustment scheme, namely, the optimal solid curve such that the representative household's goal function (1) is maximized.

[20] It is consistent with the standard result that the developed and developing countries will have the same grow rates on the balance growth paths in the endogenous growth model literature.

3 Institutional Adjustment

For simplicity, I assume $g_H = 0$ throughout the paper, so $x(t) \equiv h(t), \forall t$.[21] The developing economy is a small country so that the interest rate on the international credit market, r, is exogenous and set equal to ρ. To make the analysis empirically relevant for the convergence that is significant enough, I assume $\mu > r$.[22] The international credit market is assumed to be perfect so that the developing economy is allowed to borrow to finance its institutional adjustment. Thus any optimal (hence beneficial) institutional adjustment by definition must satisfy the budget constraint.[23]

Our task is to find an optimal adjustment scheme, $\{\delta_i, T_i\}_{i=1}^{\infty}$, and an optimal time path of consumption, $c(t) \geq 0, \forall t$, to maximize (1) subject to (2), (3), with δ_0 and h_0 given, and subject to the following budget constraint:

$$\int_0^{\infty} c(t)e^{-rt}dt \leq \sum_{i=0}^{\infty}[\int_{T_i}^{T_{i+1}} h(t)e^{-rt}dt - C(\delta_i, \delta_{i+1})e^{-rt_{i+1}}], \tag{4}$$

that is, the total present value of consumption must not exceed the total present value of output net of all the adjustment costs.

Given δ_0 and h_0, the planner's problem can be rewritten as

$$V(\delta_0, h_0) \equiv \max_{\{\delta_i, t_i\}_{i=1}^{\infty}} \sum_{i=0}^{\infty}[\int_{T_i}^{T_{i+1}} h(t)e^{-rt}dt - C(\delta_i, \delta_{i+1})e^{-rt_{i+1}}], \tag{5}$$

subject to (2), (3), and that the associated adjustments must be always affordable:

$$\sum_{i=0}^{\infty}[\int_{T_i}^{T_{i+1}} h(t)e^{-rt}dt - C(\delta_i, \delta_{i+1})e^{-rT_{i+1}}] \geq 0. \tag{6}$$

The main analytical challenge lies in the fact that the optimization problem is non-stationary in the sense that there exists no fixed point for the functional equation when we write down the Bellman equation, because the institutional variable $\delta(t)$ must be monotonically moving downward in the equilibrium and the value of $\delta(t)$ only changes for a finite number of times (to be verified soon). However, we can still analyze this problem recursively. Let N denote the total number of the institutional adjustment opportunities that are available to the planner. I first set N to be a given finite number and examine the corresponding mechanics of this dynamic system. Let V_N denote the value function with a total of N adjustment opportunities. Later, I will set $N = \infty$ and explore the optimal number of the adjustment options that are actually needed. Observe that $V(\delta_0, h_0)$ in (5) must be bounded both from above and from below because $h(t) \leq 1$, $\forall t$.

[21] The extension to the case with $g_H > 0$ is quite straightforward although the consumption behavior becomes a little more complicated.

[22] It is straightforward to analyze the case when $\mu \leq r$ by following exactly the same method.

[23] When the international credit market is imperfect, the budget constraint for the institutional adjustment becomes more complicated. Foreign aid and sovereign debt may play a role. I will leave it for future research.

3.1 No Adjustment Opportunity ($N = 0$)

When $N = 0$, the convergence occurs until the learning constraint becomes binding, so

$$V_0(\delta_0, h_0) = h_0 \int_0^{\widehat{T}} e^{\mu t} e^{-rt} dt + e^{-r\widehat{T}} \int_0^{\infty} \frac{\eta}{\delta_0} e^{-rt} dt,$$

where $\widehat{T}$ is the time point when the learning constraint just binds:

$$\widehat{T}(\delta_0, h_0) = \max\{0, \frac{1}{\mu} \ln \frac{\eta}{\delta_0 h_0}\}. \tag{7}$$

Therefore,

$$V_0(\delta_0, h_0) = \begin{cases} \frac{\mu h_0}{r(\mu-r)} \left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{\mu-r}{\mu}} - \frac{h_0}{\mu-r}, & \text{if } \delta_0 h_0 < \eta \\ \frac{h_0}{r}, & \text{if } \delta_0 h_0 \geq \eta \end{cases}. \tag{8}$$

To focus on the more interesting case, the initial human capital gap is assumed to be sufficiently large so that institutional adjustment is needed:

Assumption A0:

$$h_0 < \eta/\delta_0. \tag{A0}$$

3.2 One Adjustment Opportunity ($N = 1$)

Let T_1 denote the time when the institutional barrier is adjusted. The control can be exercised either weakly before the learning constraint binds, in which case $T_1 \leq \widehat{T}$ and we denote the value function by $G_1(\delta_0, h_0)$, or after the learning constraint binds, in which case $T_1 \geq \widehat{T}$ with value function denoted by $F_1(\delta_0, h_0)$. Therefore, $V_1(\delta_0, h_0) = \max\{G_1(\delta_0, h_0), F_1(\delta_0, h_0)\}$,where

$$G_1(\delta_0, h_0) \equiv \max_{T_1 \leq \widehat{T}, \delta_1} \int_0^{T} h_0 e^{\mu t} e^{-rt} dt + e^{-rT_1} \left[V_0(\delta_1, h_0 e^{\mu T}) - C(\delta_0, \delta_1)\right], \tag{9}$$

and

$$F_1(\delta_0, h_0) \equiv \max_{\widehat{T} \leq T_1, \delta_1} \left[\begin{array}{c} \int_0^{\widehat{T}} h_0 e^{\mu t} e^{-rt} dt + \int_{\widehat{T}}^{T_1} \frac{\eta}{\delta_0} e^{-rt} dt \\ +e^{-rT_1} \left[V_0(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1)\right] \end{array} \right]. \tag{10}$$

The following lemma says that $F_1(\delta_0, h_0) \geq G_1(\delta_0, h_0)$:

Lemma 1 $V_1(\delta_0, h_0) = F_1(\delta_0, h_0)$ *for any* (δ_0, h_0) *that satisfies Assumption A0.*

Proof. Refer to Appendix 1. ■

The intuition is straightforward. For any adjustment made strictly before the learning constraint becomes binding, the net value can be strictly increased if the same amount of adjustment is made at $\widehat{T}$. This is because the gross benefit of any such adjustment is independent of the adjustment time before the learning constraint binds and the payment of the same adjustment cost is now delayed. This lemma allows us to focus on the adjustment made only weakly after the learning barrier becomes binding. The status quo is assumed to be maintained if the net benefit of the reform is zero.

Lemma 2 *$T_1^* = \widehat{T}$ if $\delta_1^* < \delta_0$ and $T_1^* < \infty$.*

Proof. Refer to Appendix 2. ■

Lemma 2 states that the barrier, if reduced, must occur at the time when the learning constraint just becomes binding. An interior solution for the optimal adjustment size will be obtained if and only if

$$A\phi r \leq \frac{\eta}{\delta_0} \leq (A\phi r)^{\frac{1}{\phi+\frac{r}{\mu}}} , \tag{11}$$

where the first strict inequality ensures that the adjustment, if made, must be downward and the second inequality ensures the institutional barrier be no smaller than η after the adjustment. For the convenience of exposition, define

$$\widetilde{B}(z) \equiv A\left[\frac{A\phi r z}{\eta}\right]^{\frac{-\phi}{\phi+\frac{r}{\mu}-1}}\left(\frac{\phi\mu}{\mu-r}-1\right)-\frac{\eta\mu}{r(\mu-r)z};$$
$$\widehat{B}(z) \equiv \frac{\mu}{r(\mu-r)}\left(\frac{\eta}{z}\right)^{\frac{r}{\mu}}-A\left(\frac{z}{\eta}\right)^{\phi}-\frac{\eta\mu}{r(\mu-r)z}.$$

Lemma 3 *Suppose $N = 1$. When (11) is satisfied and $B < \widetilde{B}(\delta_0)$, an optimal downward barrier adjustment will be made at $\widehat{T}$ and $\delta_1^* = \theta(\delta_0)\delta_0$, where adjustment size is given by*

$$\theta(\delta_0) \equiv \left[\frac{A\phi r\delta_0}{\eta}\right]^{\frac{1}{\phi+\frac{r}{\mu}-1}} . \tag{12}$$

When $A\phi r \leq (A\phi r)^{\frac{1}{\phi+\frac{r}{\mu}}} < \frac{\eta}{\delta_0}$ and $B < \widehat{B}(\delta_0)$ hold , an optimal downward barrier adjustment will be made at $\widehat{T}$ and $\delta_1^* = \eta$. Otherwise no adjustment will be made. Corre-

spondingly, the value function is given by

$$V_1(\delta_0, h_0) = \begin{cases} \frac{\mu\eta}{r(\mu-r)}\left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{-r}{\mu}}\theta(\delta_0)^{\frac{r}{\mu}-1}\delta_0^{-1} & \textit{when} \quad \widetilde{B}(\delta_0) > B \textit{ and} \\ -\frac{h_0}{\mu-r} - \left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{-r}{\mu}}(A\theta(\delta_0)^{-\phi} + B), & \quad\quad \textit{(11) is satisfied.} \\ \frac{\mu}{r(\mu-r)}(h_0)^{\frac{r}{\mu}} - \frac{h_0}{\mu-r} & \textit{when} \quad \widehat{B}(\delta_0) > B \textit{ and} \\ -\left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{-r}{\mu}}(A(\frac{\eta}{\delta_0})^{-\phi} + B), & \quad\quad A\phi r \le (A\phi r)^{\frac{1}{\phi+\frac{r}{\mu}}} < \frac{\eta}{\delta_0} \\ V_0(\delta_0, h_0), & \quad\quad \textit{otherwise} \end{cases} . \tag{13}$$

Proof. Refer to Appendix 3. ■

An intuitive way to understand the above lemma is to see the option value of having one adjustment opportunity. Suppose the current institutional variable is δ and the learning constraint is already binding hence the current human capital gap is $x = \frac{\eta}{\delta}$. The current net gain by adjusting δ to a value $\widetilde{\delta} \in [\eta, \delta)$ is given by $V_0(\widetilde{\delta}, \frac{\eta}{\delta}y) - C(\delta, \widetilde{\delta}) - V_0(\delta, \frac{\eta}{\delta})$. Let $y \equiv \frac{\delta}{\widetilde{\delta}}$ denote the adjustment size and let $\Omega(y, \delta)$ denote the current value of the net gain by undertaking an adjustment with size y from δ. Then we have

$$\Omega(y, \delta) \equiv \frac{\mu\eta}{\delta r(\mu - r)}\left[y^{1-\frac{r}{\mu}} - 1\right] - \left[Ay^{\phi} + B\right]. \tag{14}$$

The adjustment will be exercised if and only if there exists some $\widehat{y} \in (1, \frac{\delta}{\eta}]$ such that $\Omega(\widehat{y}, \delta) > 0$. The option value of having one adjustment opportunity is $\max\{0, \max_{y \in (1, \frac{\delta}{\eta}]} \Omega(y, \delta)\}$. Let $\widetilde{y}$ denote the smallest positive root of $\Omega(y, \delta) = 0$ for given δ if there exists some $\widehat{y} \in (1, \frac{\delta}{\eta}]$ such that $\Omega(\widehat{y}, \delta) > 0$.

Now imagine there are only N adjustment opportunities available, so after the first $N - 1$ adjustment opportunities have been used, the social planner is left with only one single option to make adjustment. Taking the insitutional barrier vairable after the first $N - 1$ adjustment δ^*_{N-1} as given, this problem can be illustrated in the following figure.

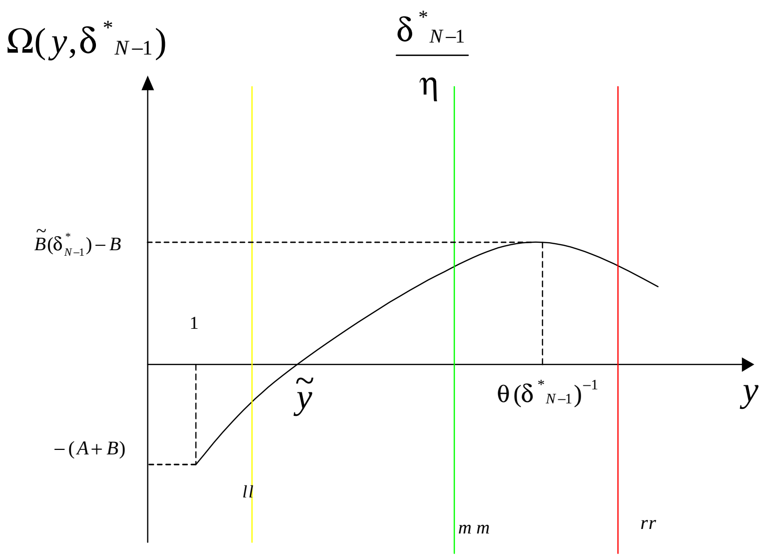

Figure 2. Last Adjustment Option

Suppose δ^*_{N-1} is small enough such that $\frac{\delta^*_{N-1}}{\eta} < \widetilde{y}(\delta^*_{N-1})$, that is, the vertical line $y = \frac{\delta^*_{N-1}}{\eta}$ is in a position such as the left vertical line ll in Figure 2, then the net benefit from this adjustment $\Omega(\frac{\delta^*_{N-1}}{\eta}, \delta^*_{N-1}) < 0$, hence it is optimal to waive the last option of adjustment. In that case, the long run steady state of the institutional variable is δ^*_{N-1}. If $\widetilde{y}(\delta^*_{N-1}) < \frac{\delta^*_{N-1}}{\eta} \leq \theta(\delta^*_{N-1})^{-1}$(see the vertical line mm), then the last adjustment is made, after which $\delta^*_N = \eta$, so ultimately the developing country will have the same human capital stock and GDP as the developed country. If $\frac{\delta^*_{N-1}}{\eta} > \theta(\delta^*_{N-1})^{-1}$(see the right vertical line rr) , then the last adjustment is also made, and the ultimate barrier variable is equal to $\delta^*_N = \theta(\delta^*_{N-1})\delta^*_{N-1}$, which is larger than η. In this case, a permanent GDP gap would exist in the long run between the less development country and the developed country. This is because the barrier adjustment becomes increasingly costly while the benefit for the same adjustment size decreases as the economy grows. At certain point, the human capital gap becomes so small that further barrier adjustment becomes unattractive.

In particular, when $N = 1$, the first inequality in (11) ensures $\theta(\delta_0)^{-1}$ is greater than one, while the second weak inequality guarantees that $\theta(\delta_0)^{-1}$ is to the left of the vertical line $y = \frac{\delta_0}{\eta}$ (in the position like rr). $B < \widetilde{B}\ (\delta_0)$ guarantees that the option value of the one adjustment opportunity be strictly positive. When the second weak inequality in (11) is violated, there are two possibilities. The adjustment will be made if line $y = \frac{\delta_0}{\eta}$ is at the position like mm, since the option value is positive. If $\frac{\delta_0}{\eta}$ is at the position like ll, then no adjustment will be made as the option value is zero. This completes the characterization for $N = 1$.

The conditions in (13) look complicated. Alternatively, the following lemma gives a useful and easy-to-check necessary condition to exercise the one-shot control.

Lemma 4 *The one-time control will be exercised only if*

$$A + B < \frac{1}{r}\left[\frac{\mu}{r}\right]^{\frac{r}{r-\mu}}, \tag{15}$$

and $\frac{\eta}{\delta_0} \in (\underline{\beta}, \overline{\beta})$ *, where* $\underline{\beta}$ *and* $\overline{\beta}$ *are the two distinct roots of the following equation:*

$$x^{\frac{r}{\mu}} = x + \frac{(A+B)\,r(\mu - r)}{\mu}.$$

Proof. Refer to Appendix 4. ■

The intuition is the following. Since any nontrivial adjustment at leat costs $A + B$, it has to be sufficiently small in order to justify the reform. Moreover, if $\frac{\eta}{\delta_0} \geq \overline{\beta}$, then δ_0 is sufficiently close to η so that the benefit from any further adjustment is too small to warrant further adjustment. If $\frac{\eta}{\delta_0} \leq \underline{\beta}$, no further one-step adjustment will be made because δ_0 is so high that it requires a large reduction in δ to achieve any given amount of utility improvement, which makes the associated adjustment cost larger than the gain from any one-step adjustment.

3.3 Optimal Adjustment

Suppose there are N adjustment options, where $1 \leq N < \infty$, then we have

$$V_N(\delta_0, h_0) = \max\{G_N(\delta_0, h_0), F_N(\delta_0, h_0)\},$$

where

$$G_N(\delta_0, h_0) \equiv \max_{T_1 \leq \widehat{T}, \delta_1} \int_0^{T_1} h_0 e^{\mu t} e^{-rt} dt + e^{-rT_1}\left[V_{N-1}(\delta_1, h_0 e^{\mu T_1}) - C(\delta_0, \delta_1)\right];$$

$$F_N(\delta_0, h_0) \equiv \max_{\widehat{T} \leq T_1, \delta_1} e^{-rT_1}\left[V_{N-1}(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1) - V_0(\delta_0, \frac{\eta}{\delta_0})\right] + \frac{h_0 \mu}{\mu - r}\left(\frac{\eta}{\delta_0 h_0}\right)^{1-\frac{r}{\mu}} - \frac{h_0}{\mu - r}.$$

Like before, $G_N(\delta_0, h_0)$ denotes the value function when the first reform occurs before the first learning constraint binds and $F_N(\delta_0, h_0)$ denotes the value function when the first reform occurs after the first learning contraint binds. By applying the recursive method, we have the following proposition.

Proposition 5 *Any institutional adjustment must occur precisely when the learning constraint just becomes binding, that is,*

$$T_i^* = \frac{1}{\mu} \ln \frac{\eta}{\delta_{i-1}^* h_0}, \forall i = 1, 2, ..., N. \tag{16}$$

In additon, the less developed economy keeps catching up at a constant speed μ until the last learning constraint just binds, after which it grows at the same speed as the developed country, that is

$$\frac{dx(t)/dt}{x(t)} = \begin{cases} \mu, & if \quad t \leq T_N^* \\ 0, & otherwise \end{cases} .$$

Proof. Refer to Appendix 5. ■

This result looks simple or even boring, but it explicitly tells us that a sequential process of cumulative institutional buildings is undergoing along the economic convergence. Under the model assumptions, the institutional barrier is sequentially reduced in a timely manner to ensure a generically unbinding learning constraint. Consequently, the GDP dynamics appear to be solely determined by the human capital accumulation, as is consistent with the standard growth literature. However, the cumulative institutional building plays a crucially important hidden role in sustaining this convergence process. Without the timely relaxations of institutional binding constraints at different development levels, the convergence would have stopped prematurely. This point is largely ignored in the existing growth literature. In addition, recall thegeneral idea in the "Washington Consensus" emphasizes that all the reforms should be undertaken in one step to ensure that all the future growth will be free of any binding institutional bottlenecks (see, for example, Stiglitz,1998). It is also often argued that gradual and partial reforms may creative more distortions, so reforms should be comprehensive and quick (see for example, Bruno, 1972; Murphy, Shleifer and Vishny, 1992). By contrast, the model developed here formalizes a rationale for why optimal reforms can be done sequentially by alleviating the different binding growth bottlenecks at different development levels. This seems more consistent with the real-life episodes of economic accelerations and reforms as summarized by Rodrik (2005) and many others (see, World Bank, 2008). It should be viewed as complementary to the existing literature on the gradual reforms with political feasibility (see, for example, the discussion on China's reform by Lau, Qian and Roland, 2000; Roland, 2002).[24]

We have shown that cumulative reforms can sustain the convergence, but can convergence last for ever such as the asymptotic approaching as predicted in the standard growth models? Or equivalently, is there going to be an infinite number of institutional adjustments? The answer is no.

Proposition 6 *There is only a finite number of institutional adjustments and thus convergence may stop ($N^* < \infty$).*

Proof. Refer to Appendix 6. ■

Notice that this proposition holds even if the fixed adjustment cost B equals zero. This proposition enables us to assume, without loss of generality, that the number of

[24]Section 4 briefly discusses what happens when some of the explicit assumptions in this normative model are not satisfied.

adjustment opportunities N is finite. Define $N^* \equiv \min \left\{ \underset{N \leq \overline{N}(\delta_0, h_0)}{\arg\max} V_N(\delta_0, h_0) \right\}$, the optimal minimum number of institutional adjustments. How to characterize N^*? Suppose $N^* \geq 1$, the original optimization problem (5) can be rewritten as

$$\max_{N, \{\delta_i\}_{i=1}^N} V_0(\delta_N, h_0) - \sum_{i=1}^{N} e^{-rT_i} \left[A \left(\frac{\delta_{i-1}}{\delta_i} \right)^{\phi} + B \right] \tag{17}$$

subject to

$$\delta_i < \delta_{i-1} \text{ for each } i = 1, ..., N,$$
$$\delta_N \geq \eta; \;\; \delta_0 \text{ and } h_0 \text{ are given,}$$

where the optimal adjustment times are given by

$$T_i = \frac{1}{\mu} \ln \frac{\eta}{\delta_{i-1} h_0}, \forall i = 1, 2, ..., N.$$

Recall that the optimal adjustment plan by definition automatically satisfies the budge constraint because of the complete international credit market. Substituting (16) into (17) and using (8), we obtain the following equivalent problem:

$$\max_{N, \{\delta_i\}_{i=1}^N} \frac{\mu \eta}{r(\mu - r)} (\delta_N)^{\frac{r}{\mu} - 1} - \sum_{i=1}^{N} \delta_{i-1}^{\frac{r}{\mu}} \left[A \left(\frac{\delta_{i-1}}{\delta_i} \right)^{\phi} + B \right]. \tag{18}$$

First observe that N^* and $\{\delta_i^*\}_{i=1}^{N^*}$ are independent of h_0 so long as assumption A0 is satisfied. The intuition is straightforward: for any given δ_0, no matter what h_0 is, the economy will have the same GDP at time $\widehat{T}$. From that point on, the optimization problem is identical and independent of h_0, so the initial institutional barrier δ_0 alone will determine the optimal adjustment scheme. Suppose $N^* \geq 2$.[25] Let $y_i \equiv \frac{\delta_{i-1}}{\delta_i}$ denotes the size of the ith institutional adjustment for $i = 1, 2, ..., N - 1$. We have the following proposition.

Proposition 7 (0) *When $A > 0$, for any $\forall i = 1, 2, ..., N^* - 1$, we must have*

$$\left(\frac{y_{i+1}}{y_i} \right)^{\phi} = \frac{y_i^{\frac{r}{\mu}} \phi - \frac{B}{A} \frac{r}{\mu}}{\frac{r}{\mu} + \phi} \text{ and } y_i > \left[\frac{rB}{A \mu \phi} \right]^{\frac{\mu}{r}}. \tag{19}$$

[1]When $A = 0$, there must be an immediate full adjustment if any adjustment is made: $N^* = 1$ and $\delta_N^* = \eta$. [2] When $B = 0$ and $A > 0$, the adjustment size is strictly monotonic or constant over time when $N^* \geq 2$. More precisely, when $\delta_{N^*}^* > \eta$,[26] we must have

$$\frac{y_{i+1}}{y_i} > 1, \forall i = 1, 2, ..., N^* - 1, \textit{ iff } y_j > \left(\frac{\frac{r}{\mu} + \phi}{\phi} \right)^{\frac{\mu}{r}}, \textit{ for some } j \in \{1, 2, ..., N^* - 1\}, \tag{20}$$

[25] In Appendix, we provide a full characterization when there are only two opportunities to adjust institutions.

[26] Equivalently, it requires $A\phi r < \left(\frac{r + \mu\phi}{\mu\phi} \right)^{\frac{-\mu(\phi-1)}{r}}$.

and

$$\frac{y_{i+1}}{y_i} = 1, \forall i = 1, 2, ..., N^* - 1, \text{ iff } y_j = \left(\frac{\frac{r}{\mu} + \phi}{\phi}\right)^{\frac{\mu}{r}}, \text{ for some } j \in \{1, 2, ..., N^* - 1\},$$

in which case

$$N^* = \frac{\log \frac{A\phi r \delta_0}{\eta}}{\frac{\mu}{r} \log\left(\frac{\frac{r}{\mu}+\phi}{\phi}\right)} + (\phi - 1), \tag{21}$$

and

$$\delta^*_{N^*} = \frac{\eta}{A\phi r}\left(\frac{r + \mu\phi}{\mu\phi}\right)^{\frac{-\mu(\phi-1)}{r}}. \tag{22}$$

When $\delta^*_{N^*} = \eta$, the adjustment is also strictly monotonic or constant over time.

Proof. Refer to Appendix 7. ■

Part [0] of this proposition states that there is a common lower bound for all the adjustment sizes. In addition, a larger $\frac{B}{A}$ implies a larger lower bound for the adjustment size. In particular, (19) states how the adjustment sizes of any two consecutive institutional reforms are related. Holding the current adjustment size and other things constant, (31) implies that a larger $\frac{B}{A}$ leads to a smaller adjustment size in *all* the future reforms. This result is intuitive: when the variable adjustment cost becomes relatively important, it is better to shrink the size of adjustment size (and also make the reforms more frequent). So if the less developed economy has a very powerful single-party administrative central government (think about China), then it has a relatively small B but a relatively big A. The model implies that the optimal reform for this economy should be more piece-meal. Even within democracies, a proportional representation parliamentary system or a presidential system can be very different from a presidential system.[27] The latter tends to have a smaller B than a parliament system, based on which this model also predicts that a democracy country with Presidential system should adopt a more gradual small-step reform than the countries with proportional representation system, holding everything else equal. By contrast, the more politically unstable countries tend to have a relatively large B, so they should adopt more radical reforms.

Part [1] of the proposition is intuitive: the reform, if initiated, should be undertaken thoroughly once and for all when the total reform cost is independent of the adjustment size. Part [2] states that, when the fixed adjustment cost is negligible, the adjustment size must change monotonically over time. In particular, suppose a GDP gap persists between the two countries (namely, $\frac{h(t)}{H(t)} < 1$ after $t \geq T_{N^*}$). Then the GDP gap is smaller than $A\phi r \left(\frac{r+\mu\phi}{\mu\phi}\right)^{\frac{\mu(\phi-1)}{r}}$ if the adjustment size is strictly increasing over time; the gap is equal to $A\phi r \left(\frac{r+\mu\phi}{\mu\phi}\right)^{\frac{\mu(\phi-1)}{r}}$ if the adjustment size is constant over time, and smaller than

[27] See, Grossman and Helpman, 2001; Persson and Tabellini, 2003.

$A\phi r\left(\frac{r+\mu\phi}{\mu\phi}\right)^{\frac{\mu(\phi-1)}{r}}$ if the adjustment size is decreasing over time. In particular, when the adjustment size is constant, the optimal number of reforms is given by (21). It implies that for a given adjustment size, the larger the variable adjustment cost, the more reforms there will be ($\frac{\partial N^*}{\partial \phi} > 0$; $\frac{\partial N^*}{\partial A} > 0$). In addition, the further away the initial institution variable is from the ideal institution η, the more reforms there will be ($\frac{\partial N^*}{\partial\left(\frac{\delta_0}{\eta}\right)} > 0$). These results are very intuitive, indicating that the less distortive the reform process (smaller ϕ or A), the fewer steps of reforms are needed.

All these predictions are unambiguous and are potentially testable empirically.

4 Further Discussion

By construction, all the reforms are the first best and therefore must be always desirable in this model. However, like the two fundamental welfare theorems, the importance of this normative investigation also lies in that it compels us to be fully aware of the whole array of the underlying assumptions that can support the first-best equilibrium: perfect credit market, perfect foresight (or no uncertainty) and sufficient capability of the institutional builder, benevolence of the institutional builder/reformer, none of which would always hold in the reality. The benchmark developed here may hopefully help us organize all these prerequisites in a logically consistent way, and thus prepare us for further exploration to incorporate more realistic considerations. Let us now discuss very briefly some possible extensions based on this model.

The assumption of international credit market completeness allows us to essentially ignore the budget constraint in the equilibrium. If the credit market is not complete so that the country has to rely on its own resources currently available to finance its institutional adjustment. Then the feasibility constraint (6) is changed to

$$\int_0^{T_{i+1}} h(t)e^{-rt}dt - \sum_{j=0}^{i} C(\delta_j, \delta_{j+1})e^{-rT_{ji+1}} \geq 0 \text{ for any } i = 0, 1, 2, ...$$

Therefore it is possible that some first-best institutional adjustment obtained in the previous section becomes no longer feasible. Consequently, it may generate a different GDP dynamics such as intermittent convergence rather than continuous convergence. It also opens the door for a nontrivial discussion on the role of foreign aid or sovereign debt. Also, to fully address issue of the fiscal feasibility, we may also explicitly introduce fiscal or monetary policies. Recall it is implicitly assumed that a lump-sum tax is imposed in this model. If, instead, only distorting tax instruments are available, then we need to derive a dynamic tax scheme and the institutional adjustment scheme simultaneously under different capital market constraint or information structures.

A second extension is to introduce uncertainty so that when the institutional bottlenecks will become binding cannot be perfectly predicted before hand. For example,consider the following dynamic process for the GDP gap between the two countries:

$$dx(t) = I(x,t)dt + \sigma(x,t)dW,$$

where W is a standard Weiner process. The model presented before is a special case with $I(x,t)$ being a step function as (2) and without uncertainty $\sigma(x,t) = 0$. Now the growth rate of the less developed economy would fluctuate stochastically and might be even negative. A major analytical difficulty is that the standard Hamilton-Jacoby-Bellman equation may not be easily formulated because human capital can not jump by its economic nature and thus it is not a control variable. This is also one of the key differences from the standard investment models. Recall my model studies the "capacity adjustment" instead of "capital adjustment" while the standard (S,s) inventory models examine how to adjust the inventory "corn flow" under the fixed capacity constraint of the "barn", but here the size of the "barn" is optimally adjusted and "the corn" comes in automatically. In a technology adoption paper by Parente (1994), the cost of technology updating comes from the instantaneous loss of expertise associated with the current technology, so what is adjusted is the technology level, similar to the (S,s) model, instead of the "capacity" that allows for technology adoption. Another important technical difference between Parente's model and mine is the budget constraint. In his model the adjustment cost is the loss of expertise thus is not paid explicitly with financial resources, hence the budget constraint problem is sidestepped. By contrast, the budget constraint becomes crucial, especially when the credit market is incomplete in my model.

A third possible avenue is to take a positive approach and embed more explicit political-economic micro-foundations for the process of reforms. Either the assumption of the central government's benevolence is relaxed, or we may formulate a social choice problem for the process of reaching a compromised reform plan with multiple political players. Commitment and time consistency may become very important issues as argued by Acemoglu *et al* (2005). Moreover, governments are certainly not omnipotent and may sometimes adopt wrong reform recipes or growth strategies (Stiglitz, 1998; Rodrik, 2005; World Bank, 2008; Lin, 2009).

5 Concluding Remarks

Ample empirical evidences suggest that economic accelerations are sustained only by sequentially relaxing different institutional barriers that strangle economic growth at different development stages. But there is a paucity of theoretical models that capture this important fact in the growth literature. This paper aims to fill this gap by providing a theoretical model to explain how economic convergence interacts with endogenous institutional adjustment. I characterize how the social planner in a developing economy *should* optimally adjust its institutional barrier and how this endogenous institutional building affects the catching up behavior. The model may help us understand normatively the interactive mechanics of the first-best institutional adjustment and the associated economic growth. Hopefully, this simple model may also prove useful when incorporating more realistic and complicated factors into the analysis in the future. Another fruitful direction for future research is to conduct careful empirical investigations to assess how far the real world deviates from this first-best scenario, and help identify which implicit assumption is the key source that drives the potential discrepancy between the model predictions and the reality.

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Appendix 1. Proof of Lemma 1: By contradiction, suppose there exists an optimal adjustment time $T_1^* \in [0, \widehat{T})$ and a real adjustment is made so that $\delta_1 \neq \delta_0$. Substituting (8) into (9), we can easily prove that for $\forall T_1 \in (0, \widehat{T}]$,

$$\frac{\partial}{\partial T_1}\left\{\int_0^{T_1} h_0 e^{\mu t} e^{-rt} dt + e^{-rT_1}\left[V_0(\delta_1, h_0 e^{\mu T_1}) - C(\delta_0, \delta_1)\right]\right\} > 0, \forall \delta_1 \neq \delta_0.$$

Moreover, any adjustment affordable at T_1^* must be affordable at $\widehat{T}$. Contradiction. If it's optimal not to make any adjustment, $F_1(\delta_0, h_0) = V_0(\delta_0, h_0)$, where $T_1^* = \infty$. Q.E.D

Appendix 2. Proof of Lemma 2: The previous lemma shows $T_1^* \geq \widehat{T}$. When some nontrivial adjustment is made ($\delta_1^* < \delta_0 and T_1^* < \infty$), we must have $V_0(\delta_0, \frac{\eta}{\delta_0}) < V_0(\delta_1^*, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1^*)$. By Lemma 1 and the functional form of V_0 in (8), we must have

$$\frac{\partial\left[\text{RHS of } F_1(\delta_0, h_0)\right]}{\partial T_1} = re^{-rT_1}[V_0(\delta_0, \frac{\eta}{\delta_0}) + C(\delta_0, \delta_1) - V_0(\delta_1, \frac{\eta}{\delta_0})] \leq 0.$$

Since $T_1^* < \infty$, it must be that $\frac{\partial[\text{RHS of } F_1(\delta_0, h_0)]}{\partial T_1} < 0$ hence $T_1^* = \widehat{T}$.Q.E.D

Appendix 3. Proof of Lemma 3: We have the following two first order conditions with respect to T_1 and δ_1 :

$$\frac{\partial\left[\text{RHS of } F_1(\delta_0, h_0)\right]}{\partial T_1} = re^{-rT_1}[V_0(\delta_0, \frac{\eta}{\delta_0}) + C(\delta_0, \delta_1) - V_0(\delta_1, \frac{\eta}{\delta_0})] \leq 0$$
$$\frac{\partial\left[\text{RHS of } F_1(\delta_0, h_0)\right]}{\partial \delta_1} = 0 \Rightarrow \delta_1^* = \theta(\delta_0)\delta_0 \in [\eta, \delta_0) \text{ guaranteed by A1.}$$

The left inequality of (11) guarantees that $\delta_1^* < \delta_0$, while the right weak inequality of (11) makes sure that $\delta_1^* \geq \eta$. The condition $B < \widetilde{B}(\delta_0)$ ensures

$$V_0(\delta_0, \frac{\eta}{\delta_0}) + C(\delta_0, \delta_1^*) - V_0(\delta_1^*, \frac{\eta}{\delta_0}) < 0,$$

therefore $T_1^* = \widehat{T}$. The second order condition is also satisfied. Under A0 , (11), and $B < \widetilde{B}(\delta_0)$, we have

$$\begin{aligned} V_1(\delta_0, h_0) = & \frac{\mu\eta}{r(\mu - r)}\left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{-r}{\mu}} \theta(\delta_0)^{\frac{r}{\mu}-1}\delta_0^{-1} \\ & - \frac{h_0}{\mu - r} - \left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{-r}{\mu}} (A\theta(\delta_0)^{-\phi} + B), \end{aligned} \tag{23}$$

where $\theta(\delta_0)$ is given by (12). If $B > \widetilde{B}(\delta_0)$ or if the left inequality in (11) is violated, then $V_1(\delta_0, h_0) = V_0(\delta_0, h_0)$. If the right weak inequality in (11) is violated, then there are two possibilities. One is to waive the adjustment option because the adjustment cost dominates even the largest possible gain from an institutional adjustment, in which case we have

$$V_1(\delta_0, h_0) = V_0(\delta_0, h_0) \text{ if } V_0(\delta_0, \frac{\eta}{\delta_0}) + C(\delta_0, \eta) - V_0(\eta, \frac{\eta}{\delta_0}) \geq 0.$$

The second possibility is to exercise the adjustment option by fully exhausting the learning potential:

$$\delta_1^* = \eta \ \text{ and } V_0(\delta_0, \frac{\eta}{\delta_0}) + C(\delta_0, \eta) - V_0(\eta, \frac{\eta}{\delta_0}) < 0.$$

The second possibility requires $\widehat{B}(\delta_0) > B$, where

$$\widehat{B}(y) \equiv \frac{\mu}{r(\mu - r)} \left(\frac{\eta}{y}\right)^{\frac{r}{\mu}} - A \left(\frac{y}{\eta}\right)^{\phi} - \frac{\eta\mu}{r(\mu - r)y}.$$

In that case,

$$V_1(\delta_0, h_0) = \frac{\mu}{r(\mu - r)} (h_0)^{\frac{r}{\mu}} - \frac{h_0}{\mu - r} - \left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{-r}{\mu}} (A(\frac{\eta}{\delta_0})^{-\phi} + B).$$

In summary, the value function with one adjustment opportunity is given by

$$V_1(\delta_0, h_0) = \begin{cases} \frac{\mu\eta}{r(\mu-r)} \left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{-r}{\mu}} \theta(\delta_0)^{\frac{r}{\mu}-1}\delta_0^{-1} - \frac{h_0}{\mu-r} - \left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{-r}{\mu}} (A\theta(\delta_0)^{-\phi} + B), & \text{when } \widetilde{B}(\delta_0) > B \text{ and (11) is satisfied.} \\ \frac{\mu}{r(\mu-r)} (h_0)^{\frac{r}{\mu}} - \frac{h_0}{\mu-r} - \left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{-r}{\mu}} (A(\frac{\eta}{\delta_0})^{-\phi} + B), & \text{when } \widehat{B}(\delta_0) > B \text{ and } A\phi r \leq (A\phi r)^{\frac{1}{\phi + \frac{r}{\mu}}} < \frac{\eta}{\delta_0} \\ V_0(\delta_0, h_0), & \text{otherwise} \end{cases}.$$

Q.E.D.

Appendix 4. Proof of Lemma 4: The one-time control will be exercised non-trivially if and only if

$$V_0(\delta_0, \frac{\eta}{\delta_0}) < V_0(\delta, \frac{\eta}{\delta_0}) - C(\delta_0, \delta),$$

so $\delta < \delta_0$ only if

$$V_0(\delta_0, \frac{\eta}{\delta_0}) < V_0(\delta, \frac{\eta}{\delta_0}) - (A + B),$$

which implies (recall $\mu > r$)

$$\delta < \left[\delta_0^{\frac{r}{\mu}-1} + \frac{\delta_0^{\frac{r}{\mu}} (A + B)\, r(\mu - r)}{\mu\eta}\right]^{\frac{1}{\frac{r}{\mu}-1}}.$$

Since $\delta \geq \eta$, we require

$$\eta < \left[\delta_0^{\frac{r}{\mu}-1} + \frac{\delta_0^{\frac{r}{\mu}} (A + B)\, r(\mu - r)}{\mu\eta}\right]^{\frac{1}{\frac{r}{\mu}-1}},$$

or equivalently,

$$\left(\frac{\eta}{\delta_0}\right)^{\frac{r}{\mu}} > \frac{\eta}{\delta_0} + \frac{(A + B)\, r(\mu - r)}{\mu},$$

which is possible if and only if (15) is satisfied and $\frac{\eta}{\delta_0} \in (\underline{\beta}, \overline{\beta})$. (15) ensures $0 < \underline{\beta} < \left[\frac{\mu}{r}\right]^{\frac{1}{\frac{r}{\mu}-1}} < \overline{\beta} < 1$.Q.E.D.

Appendix 5. Proof of Proposition 1:

$$V_N(\delta_0, h_0) = \max\{G_N(\delta_0, h_0), F_N(\delta_0, h_0)\}, \tag{24}$$

where

$$\begin{aligned} G_N(\delta_0, h_0) &\equiv \max_{T_1 \le \widehat{T}, \delta_1} \int_0^{T_1} h_0 e^{\mu t} e^{-rt} dt + e^{-rT_1} \left[V_{N-1}(\delta_1, h_0 e^{\mu T_1}) - C(\delta_0, \delta_1)\right]; \\ F_N(\delta_0, h_0) &\equiv \max_{\widehat{T} \le T_1, \delta_1} e^{-rT_1} \left[V_{N-1}(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1) - V_0(\delta_0, \frac{\eta}{\delta_0})\right] \\ &\quad + \frac{h_0 \mu}{\mu - r} \left(\frac{\eta}{\delta_0 h_0}\right)^{1-\frac{r}{\mu}} - \frac{h_0}{\mu - r}. \end{aligned}$$

First observe

$$\begin{aligned} G_N(\delta_0, h_0) &\equiv \max_{T_1 \le \widehat{T}, \delta_1} \int_0^{T_1} h_0 e^{\mu t} e^{-rt} dt + e^{-rT_1} V_{N-1}(\delta_1, h_0 e^{\mu T_1}) - e^{-rT_1} C(\delta_0, \delta_1) \\ &= \max_{T_1 \le \widehat{T}, \delta_1} \left[V_{N-1}(\delta_1, h_0) - e^{-rT_1} C(\delta_0, \delta_1)\right] \\ &\le \max_{\delta_1} \left[V_{N-1}(\delta_1, h_0) - e^{-r\widehat{T}} C(\delta_0, \delta_1)\right] \\ &\le F_N(\delta_0, h_0), \end{aligned}$$

hence $V_N(\delta_0, h_0) = F_N(\delta_0, h_0)$ for any $N \ge 1$. That is, no adjustment will be made before the learning barrier becomes binding. Second, no adjustment will be made strictly after the learning barrier becomes binding. This is because $F_N(\delta, x) > V_{N-1}(\delta, x)$ if and only if there exists a $\widetilde{\delta} \in [\eta, \delta]$ such that

$$[V_{N-1}(\widetilde{\delta}, x) - C(\delta, \widetilde{\delta})] - V_{N-1}(\delta, x) > 0. \tag{25}$$

That is, an adjustment will be made if and only if the net benefit from the adjustment exceeds the value without adjustment. Suppose at time t the learning barrier becomes binding (that is, $x = \frac{\eta}{\delta}$), then if $F_N(\delta, x) > V_{N-1}(\delta, x)$, it's optimal to make the adjustment without any delay because of the time discounting. Note that the left hand side of (25) is the instantaneous value of net benefit from adjustment, which determines the optimal adjustment $\widetilde{\delta}$ by the first order condition. So the instantaneous value of the net benefit from adjustment is exactly the same for any time weakly after t. Moreover, the adjustment can always be fully financed because (25) implies the adjustment is profitable. The implied GDP dynamics is obvious.Q.E.D.

Appendix 6. Proof of Proposition 2: According to the previous proposition, there will be no adjustment after $\widehat{t} = -\frac{\ln h_0}{\mu}$, the time point when the developing country exactly achieves the same human capital level as the developed country if all the potential benefit of externality can be fully exploited. The total present discounted value of the gross benefit from the whole scheme of institutional adjustment can be no larger than

$$V_0(\eta, h_0) - V_0(\delta_0, h_0) = \frac{\mu}{r(\mu - r)} \left(\frac{1}{h_0}\right)^{\frac{-r}{\mu}} - \frac{\eta}{\delta_0} \frac{\mu}{r(\mu - r)} \left(\frac{\eta}{\delta_0 h_0}\right)^{\frac{-r}{\mu}}.$$

The present discounted cost of each downward adjustment can be no smaller than

$$e^{-r\widehat{t}}(A + B).$$

The minimum optimal number of adjustments is therefore no larger than $\frac{V_0(\eta,h_0)-V_0(\delta_0,h_0)}{e^{-r\widehat{t}}(A+B)}$. This is also true even when A or B equals zero.Q.E.D.

Appendix 7. Characterization of the problem when $N = 2$.

Similar to the previous case, when two adjustment opportunities are available, the value function becomes

$$V_2(\delta_0, h_0) = \max\{G_2(\delta_0, h_0), F_2(\delta_0, h_0)\},$$

where $G_2(\delta_0, h_0)$ is the value function when the first adjustment occurs before $\widehat{T}$:

$$G_2(\delta_0, h_0) \equiv \max_{T_1 \leq \widehat{T}, \delta_1} \int_0^{T_1} h_0 e^{\mu t} e^{-rt} dt + e^{-rT_1} \left[V_1(\delta_1, h_0 e^{\mu T_1}) - C(\delta_0, \delta_1)\right],$$

and $F_2(\delta_0, h_0)$ is the value function when the first adjustment occurs after $\widehat{T}$:

$$F_2(\delta_0, h_0) \equiv \max_{\widehat{T} \leq T_1, \delta_1} \left[\begin{array}{l} \int_0^{\widehat{T}} h_0 e^{\mu_0 t} e^{-rt} dt + \int_{\widehat{T}}^{T_1} \frac{\eta}{\delta_0} e^{-rt} dt \\ +e^{-rT_1} \left[V_1(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1)\right] \end{array} \right].$$

Note that

$$\begin{aligned} G_2(\delta_0, h_0) &= \max_{T_1 \leq \widehat{T}, \delta_1} \left[V_1(\delta_1, h_0) - e^{-rT_1} C(\delta_0, \delta_1)\right] \\ &\leq \max_{\delta_1} \left[V_1(\delta_1, h_0) - e^{-r\widehat{T}} C(\delta_0, \delta_1)\right] \leq F_2(\delta_0, h_0), \end{aligned}$$

therefore $V_2(\delta_0, h_0) = F_2(\delta_0, h_0)$. Suppose two nontrivial adjustments will be made. There are again two possibilities. First, when (11) is satisfied, the first order condition with respect to δ_1 yields

$$B \frac{r}{\mu} \delta_1^{\frac{r}{\mu}+\phi} = A \delta_0^{\phi+\frac{r}{\mu}} \phi - k \delta_1^{\frac{r}{\mu}+\phi-\frac{\phi}{\frac{r}{\mu}+\phi-1}}, \tag{26}$$

where $k \equiv \frac{\eta(\phi\mu+r)}{r\phi\mu} \left(\frac{Ar\phi}{\eta}\right)^{\frac{\frac{r}{\mu}-1}{\frac{r}{\mu}+\phi-1}}$. Although no closed-form solution can be obtained, it can be easily shown that the solution exists and is unique if $\frac{r}{\mu} + \phi \geq 2$, which will be assumed for the rest of this paper. In Figure 3, the upward-sloping curve plots the term on the left hand side of (26) while the downward-sloping curve corresponds to the right hand side.

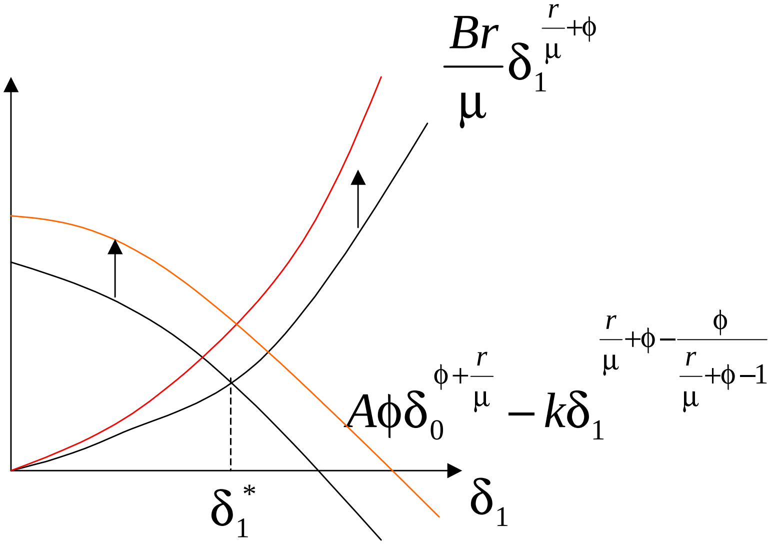

Figure 3. Optimal Adjustment Size when $N = 2$

Let δ_1^* denote the unique solution to equation (26). $\delta_1^* > \frac{\eta}{(Ar\phi)^{\frac{1}{\frac{r}{\mu}+\phi}}}$ must hold, so

$$\eta < \delta_0 \left[\frac{A^2 r \phi^2}{B\frac{r}{\mu} + \frac{(\phi\mu + r)}{r\phi\mu} (Ar\phi)^{\frac{\left(\frac{r}{\mu}-1\right)\left(\frac{r}{\mu}+\phi\right)+\phi}{\left(\frac{r}{\mu}+\phi-1\right)\left(\frac{r}{\mu}+\phi\right)}}} \right]^{\frac{1}{\frac{r}{\mu}+\phi}}. \tag{27}$$

In addition, $\delta_1^* < \delta_0$ must also hold, or equivalently,

$$A\phi - B\frac{r}{\mu} \leq 0, \text{ or } \eta < \delta_0 \left[\frac{r\phi\mu\left(A\phi - B\frac{r}{\mu}\right)}{(\phi\mu + r)(Ar\phi)^{\frac{\frac{r}{\mu}-1}{\frac{r}{\mu}+\phi-1}}} \right]^{\frac{\frac{r}{\mu}+\phi-1}{\phi}} \quad \text{if } A\phi - B\frac{r}{\mu} > 0. \tag{28}$$

Moreover, the first order condition with respect to T_1 implies that the optimal time $T_1^* = \widehat{T}$ if $V_0(\delta_0, \frac{\eta}{\delta_0}) + C(\delta_0, \delta_1^*) - V_1(\delta_1^*, \frac{\eta}{\delta_0}) \leq 0$, which is equivalent to $\Delta(\delta_0, \delta_1^*) \geq 0$ when both (27) and (28) are satisfied, where

$$\Delta(\delta_0, \delta_1^*) \equiv (\delta_1^*)^{\frac{r}{\mu} - \frac{\phi}{\frac{r}{\mu}+\phi-1}} \left(\frac{Ar\phi}{\eta}\right)^{\frac{\frac{r}{\mu}-1}{\frac{r}{\mu}+\phi-1}} \frac{\eta}{r\phi} \left(\frac{\phi\mu}{\mu - r} - 1\right) \delta_0^{-\frac{r}{\mu}}$$
$$- \frac{\eta\mu}{r(\mu - r)\delta_0} - B - B\left(\frac{\delta_1^*}{\delta_0}\right)^{\frac{r}{\mu}} - A\delta_0^{\phi}\delta_1^{*-\phi}.$$

We still need to check whether $\widetilde{B}(\delta_1^*) \geq B$. When $\Delta(\delta_0, \delta_1^*) \geq 0$ and $\widetilde{B}(\delta_1^*) \geq B$ are both satisfied, we have $\delta_2^* = \theta(\delta_1^*)\delta_1^*$ and $T_2^* = \frac{1}{\mu} \ln \frac{\eta}{\delta_1^* h_0}$. The developing economy is growing at speed $(\mu + g_H)$ up to the time point $\frac{1}{\mu}\ln\frac{\eta}{\delta_2^* h_0}$, after which the convergence stops and there will be a permanent GDP gap between the two economies ($\frac{\eta}{\delta_2^*} < 1$).

A simple comparative statics analysis gives us the following result: An increase in B will move δ_1^* leftward (see Figure 3), this is because the costing-saving motive will make adjustment less frequent but the size for each adjustment larger. In contrast, a higher A results in a higher δ_1^* because the variable adjustment cost parameter A affects the marginal adjustment cost. The higher the initial barrier, the more modest the first target of barrier adjustment.

In addition, we have $\frac{\partial \delta_1^*}{\partial \eta} < 0$, implying that the first adjustment is larger when the relative scale of the economy becomes bigger and $\frac{\partial \delta_1^*}{\partial \delta_0} > 0$, meaning the institutional barrier exhibits certain persistence as an initial inferior institution leads to an relatively inferior institution after the first adjustment.

When $\widetilde{B}(\delta_1^*) < B$ or $\Delta(\delta_0, \delta_1^*) < 0$ or any other conditions are not satisfied, the following problem needs to be solved:

$$F_2(\delta_0, h_0) \equiv \max_{\delta_1} \int_0^{\widehat{T}} h_0 e^{\mu_0 t} e^{-rt} dt + e^{-r\widehat{T}} \left[V_1(\delta_1, \frac{\eta}{\delta_0}) - C(\delta_0, \delta_1) \right],$$

where

$$V_1(\delta_1, \frac{\eta}{\delta_0}) = \frac{\mu}{r(\mu - r)} \left(\frac{\eta}{\delta_0} \right)^{\frac{r}{\mu}} - \frac{\eta}{\delta_0 (\mu - r)} - \left(\frac{\delta_0}{\delta_1} \right)^{\frac{-r}{\mu}} (A(\frac{\eta}{\delta_1})^{-\phi} + B).$$

The first order condition is

$$-(\frac{r}{\mu} + \phi) A \eta^{-\phi} \delta_1^{\phi} + \phi A \delta_0^{\phi + \frac{r}{\mu}} \delta_1^{-\phi - \frac{r}{\mu}} = B \frac{r}{\mu}, \tag{29}$$

which implies the existence and uniqueness of the root δ_1^*. So $\frac{\partial \delta_1^*}{\partial \eta} > 0$; $\frac{\partial \delta_1^*}{\partial \delta_0} > 0$; $\frac{\partial \delta_1^*}{\partial A} > 0$; $\frac{\partial \delta_1^*}{\partial B} < 0$. The only difference from the previous case is that $\frac{\partial \delta_1^*}{\partial \eta}$ has a different sign. The following two conditions need verifying:

$$\begin{aligned} &\frac{\mu}{r(\mu - r)} \left(\frac{\eta}{\delta_0} \right)^{\frac{r}{\mu}} - \frac{\frac{\eta}{\delta_0}}{\mu - r} - \left(\frac{\delta_0}{\delta_1^*} \right)^{\frac{-r}{\mu}} (A(\frac{\eta}{\delta_1^*})^{-\phi} + B) \\ &\geq C(\delta_0, \delta_1^*) + V_0(\delta_0, \frac{\eta}{\delta_0}), \end{aligned} \tag{30}$$

and $\widehat{B}(\delta_1^*) \geq B$. (29) implies that $\delta_1^* > \eta$ is equivalent to $\eta < \left[\frac{A\phi}{(A+B)\frac{r}{\mu} + A\phi} \right]^{\frac{1}{\frac{r}{\mu}+\phi}} \delta_0$. And to ensure $\delta_2^* = \eta$, we must require $\eta \geq \left[\frac{A^2\phi^2 r}{B\frac{r}{\mu} + A(\phi + \frac{r}{\mu})(A\phi r)^{-\frac{\phi}{\frac{r}{\mu}+\phi}}} \right]^{\frac{1}{\frac{r}{\mu}+\phi}} \delta_0$. It also ensures $\delta_2^* < \delta_1^*$. To summarize, we have the following result:

Suppose $N=2$ and both Assumptions A0 and (11) are satisfied. [1] $\delta_2^*=\eta$ if and only if $\left[\frac{A^2\phi^2 r}{B\frac{r}{\mu}+A(\phi+\frac{r}{\mu})(A\phi r)^{-\frac{\phi}{\frac{r}{\mu}+\phi}}}\right]^{\frac{1}{\frac{r}{\mu}+\phi}} \leq \frac{\eta}{\delta_0} < \left[\frac{A\phi}{(A+B)\frac{r}{\mu}+A\phi}\right]^{\frac{1}{\frac{r}{\mu}+\phi}}$, (30) is satisfied and $\widehat{B}(\delta_1^*)\geq B$, where δ_1^* is uniquely determined in (29); [2] $\delta_2^*>\eta$ if and only if $\Delta(\delta_0,\delta_1^*)\geq 0$, $\widetilde{B}(\delta_1^*)\geq B$, (27) and (28) are all satisfied, where δ_1^* is uniquely determined by (26) and $\delta_2^*=\theta(\delta_1^*)\delta_1^*$; [3] Otherwise, $V_2(\delta_0,h_0)=V_1(\delta_0,h_0)$ given by (13).

Appendix 8. Proof of Proposition 3: The first order condition with respect to δ_i is

$$\left(\frac{\delta_i}{\delta_{i+1}}\right)^{\phi}\left[\frac{r}{\mu}+\phi\right]+\frac{B}{A}\frac{r}{\mu}=\left[\frac{\delta_{i-1}}{\delta_i}\right]^{\frac{r}{\mu}+\phi}\phi \text{ for } \forall i<N^*, \tag{31}$$

which can be rewritten as

$$\left(\frac{y_{i+1}}{y_i}\right)^{\phi}=\frac{y_i^{\frac{r}{\mu}}\phi-\frac{B}{A}\frac{r}{\mu}}{\frac{r}{\mu}+\phi} \text{ for } \forall i<N^*,$$

which requires $y_i>\left[\frac{rB}{A\mu\phi}\right]^{\frac{\mu}{r}}$.

The first order condition with respect to δ_{N^*} is

$$\left(\frac{Ar\phi}{\eta}\delta_{N-1}^{\frac{r}{\mu}+\phi}\right)^{\frac{1}{\frac{r}{\mu}+\phi-1}}=\delta_N \text{ if } \delta_N>\eta, \tag{32}$$
$$\left(\frac{Ar\phi}{\eta}\delta_{N-1}^{\frac{r}{\mu}+\phi}\right)^{\frac{1}{\frac{r}{\mu}+\phi-1}}\leq\delta_N \text{ if } \delta_N=\eta.$$

To solve the problem completely, we define $\delta_{N-2}^*\equiv\Gamma(\ \delta_{N-1}^*,\delta_N^*)$ from (31) when $\delta_N^*>\eta$.

Obviously, $\Gamma_1>0$ and $\Gamma_2<0$. Recursively, we have

$$\delta_{N-3}^*=\Gamma(\delta_{N-2}^*,\delta_{N-1}^*)=\Gamma(\Gamma(\delta_{N-1}^*,\delta_N^*),\delta_{N-1}^*);$$
$$\delta_{N-4}^*=\Gamma(\delta_{N-3}^*,\delta_{N-2}^*)=\Gamma(\Gamma(\Gamma(\delta_{N-1}^*,\delta_N^*),\delta_{N-1}^*),\Gamma(\delta_{N-1}^*,\delta_N^*));$$
$$...$$

We ultimately have δ_0 as a function of δ_{N-1}^* and δ_N^*. Together with (32), both δ_{N-1}^* and δ_N^* , hence everything else, can be pinned down. When $\delta_N=\eta$, then $\delta_{N-2}^*\equiv\Gamma(\ \delta_{N-1}^*,\eta)$. Using the same recursive substitution, we can express δ_0 as a function of δ_{N-1}^*, from which δ_{N-1}^* hence δ_i^* can be obtained for $\forall i=1,2,...N$.

[1] When $A=0$, (18) becomes

$$\max_{N,\{\delta_i\}_{i=1}^N}\frac{\mu\eta}{r(\mu-r)}\left(\delta_N\right)^{\frac{r}{\mu}-1}-B\sum_{i=1}^{N}\delta_{i-1}^{\frac{r}{\mu}}.$$

Obviously, $N^* = 1$ if any adjustment will be made. $\delta_1^* = \eta$ is the solution for the following problem

$$\max_{\delta_1 \geq \eta} \frac{\mu\eta}{r(\mu - r)} (\delta_1)^{\frac{r}{\mu}-1} - B\delta_0^{\frac{r}{\mu}}.$$

[2] When $B = 0$,

$$\max_{N,\{\delta_i\}_{i=1}^N} \frac{\mu\eta}{r(\mu - r)} (\delta_N)^{\frac{r}{\mu}-1} - A\sum_{i=1}^{N} \delta_{i-1}^{\frac{r}{\mu}} \left(\frac{\delta_{i-1}}{\delta_i}\right)^{\phi},$$

so N^* is still finite because $\frac{\delta_{i-1}}{\delta_i} > 1$ and $\delta_i^* \geq \eta$ for any. When $N^* \geq 2$, (31) is reduced to

$$\left(\frac{\delta_i}{\delta_{i+1}}\right) = \left(\frac{\phi}{\frac{r}{\mu} + \phi}\right)^{\frac{1}{\phi}} \left(\frac{\delta_{i-1}}{\delta_i}\right)^{\frac{\frac{r}{\mu}+\phi}{\phi}}. \tag{33}$$

Denote the size of the ith institutional adjustment by $y_i \equiv \frac{\delta_{i-1}}{\delta_i}$, $i = 1, 2, ..., N$. (33) becomes

$$y_{i+1} = \left(\frac{\phi}{\frac{r}{\mu} + \phi}\right)^{\frac{1}{\phi}} y_i^{\frac{\frac{r}{\mu}+\phi}{\phi}},$$

which implies (20). The optimal adjustment size is therefore monotonic over time, either strictly increasing or strictly decreasing, or staying constant.

Lemma 3 implies that $\delta_N = \theta(\delta_{N-1})\delta_{N-1}$ when $\delta_N > \eta$, so $y_N = \frac{\delta_{N-1}}{\delta_N} = \left[\frac{A\phi r\delta_{N-1}}{\eta}\right]^{-\frac{1}{\phi+\frac{r}{\mu}-1}}$. According to (20), the adjustment size is strictly increasing if and only if $\delta_{N-1}^* < \frac{\eta}{A\phi r}\left(\frac{r+\mu\phi}{\mu\phi}\right)^{\frac{-(\mu\phi+r-\mu)}{r}}$. The adjustment size is constant if and only if $\delta_{N-1}^* = \frac{\eta}{A\phi r}\left(\frac{r+\mu\phi}{\mu\phi}\right)^{\frac{-(\mu\phi+r-\mu)}{r}}$,in which case (22) is obtained and it can be verified that $\delta_{N^*}^* > \eta$. (21) can be easily derived from (33).

When $\delta_{N^*}^* = \eta$, and $N^* \geq 2$, then by applying (29), we have

$$-(\frac{r}{\mu} + \phi)A\left[\frac{\delta_{N^*-1}}{\eta}\right]^{\phi} + \phi A\left[\frac{\delta_{N^*-2}}{\delta_{N^*-1}}\right]^{\phi+\frac{r}{\mu}} = B\frac{r}{\mu},$$

then equal adjustment size is possible only if

$$\frac{\delta_{N^*-1}}{\eta} = \frac{\delta_{N^*-2}}{\delta_{N^*-1}} = \left[\frac{(\frac{r}{\mu} + \phi)}{\phi}\right]^{\frac{\mu}{r}},$$

which is still consistent with the optimal size obtained when $\delta_{N^*}^* > \eta$. Q.E.D.