Endogenous Social Infrastructure and Economic Development

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First Version: February 2006
This Version: September 2007

Abstract: This paper explores how the distributions of political power and productivity would determine the coalition formation among different social groups and how the endogenous social infrastructure affects the growth rate and consumption inequality. By focusing on the Markov-Nash perfect equilibria with full commitment, we obtain the close-form solutions. We show that: (1) The social infrastructure of a meritocracy can be fully characterized on a simplex, indicating that a more equal distribution of political power and labor productivity will lead to more coalitions and hence a higher growth rate; (2) A minor change in the distribution of productivities (or interpreted as an adoption of a different technology) could result in a non-monotonic and drastic variation in the coalition structure and in the macroeconomic performance; (3) The ruling class in a non-democracy, even if there exists no risk of being overthrown, might voluntarily relegate some political powers to the other social groups, which could lead to growth maximization incidentally. Moreover, the existence of asymmetric equilibrium suggests that endogenous inequality might arise among the perfectly ex ante identical agents in the absence of any exogenous shocks.

Key words: Political Economy; Social Infrastructure; Endogenous Coalitions; Consumption Distribution; Economic Growth.

JEL Codes: C73, E2, O11, O40, P16

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1. Introduction

The past fifteen years have witnessed an accelerating interest in formally modeling and empirically quantifying how the social infrastructure or institutions might affect a nation's macroeconomic performance. In particular, Hall and Jones (1999) conclude that the most important factor that accounts for the huge cross-country difference in the output per worker is the social infrastructure. Acemoglu (2006) proceeds to ask why inefficient institutions can arise and persist. He argues that the institutions govern the economic policy-making process, and different economic policies lead to different economic outcomes. Hence the social groups' preferences over these different outcomes will determine their preferences over the different institutions. The social conflict determines the dynamics (and inertia) of the institutions.

This paper continues along this direction by examining more explicitly how the distributions of the political power and the labor productivity determine the coalition formation among different social groups and how the endogenous social infrastructure affects the first two moments of the macroeconomic performance, namely, the growth rate and the consumption distribution. Our analysis is most applicable to the non-democratic developing economies where the property rights might not be perfectly secure.

To gain more intuitions why it's useful to consider the macroeconomic performance through the lens of endogenous social coalitions, a quick review of China's archetypal experience is helpful. China's economic acceleration started in the late 1970s with the adoption of the household responsibility system (HRS), which tremendously increased the productivity in the agricultural production (Lin,1992). However, the initial practice of HRS would be impossible without the coalition between the farmers and the local bureaucrats, because such practice had been strictly forbidden as the “sin of capitalism” as it allowed farmers to work separately on the publicly owned land and keep the surplus after fulfilling certain quota. The local bureaucrats took the political risk by secretly supporting such practice. Another example of social coalition is the township and village enterprises, which appeared in the middle 1980s. It contributed substantially to China’s growth for more than a decade (Qian, 2003). They were semi-public firms co-managed by the grassroots entrepreneurs and local government officials. Subsequently, new coalitions between bureaucrats and the rising class of “capitalists” were manifested in the form of joint ventures or domestic private firms. The owners of these private businesses were badly in need of the strong political protection and many business opportunities provided by the government officials while the latter also benefited economically from the coalitions. It’s followed by the political power redistribution among the social groups, as was evidenced by the modifications of China’s constitution, which changed from completely prohibiting any private business to formally acknowledging that the “non-state-owned economies are indispensable ingredients of the socialist market economy”. Representatives of the non-state-owned businessmen were invited into the National Congress to participate in the policy-making processes, which reduced their conceived risk of property expropriation and hence boosted private investment and economic growth.

As the local governments gain more economic autonomy delegated by the central government and the legitimacy of private profit-seeking activities is more established, the coalitions between the local bureaucrats and the rising social class of rich entrepreneurs have been even more strengthened in recent years to further expand the private business, domestic or foreign, sometimes even at the expenses of poor people's interest. Examples include the government's mandatory takeover of the peasants' land for real estate businessmen, the black-box practice in the housing market, and irregular speculations in the stock markets caused by illegal information disclosure, etc. The partial coalition exacerbates the economic inequality in the society although the economic growth rate still appears high.[1] The social conflict has become increasingly prominent that Hu Jintao's government has announced urgently as its primary goal to build a harmonious society.

It seems to us that a deep understanding of China's three-decade episode of development requires a sufficient understanding of the underlying social infrastructure. And China is nothing but only one of many such economies that are far from being well understood. Therefore, we are motivated to develop a tractable but general theoretical model to characterize the social coalition formation and its impact on the macroeconomic performance. Different coalitions are interpreted as different social infrastructures that

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[1] China's Gini Coefficient has exceeded 0.42 for more than a decade although its annual GDP growth rate has been close to 10% for more than two decades.

determine the implicit economic policies in the economy.

Stark as it is, our model generates a set of rich implications, especially when the social groups are heterogeneous: (1) The endogenous social infrastructure of a meritocracy can be fully characterized on a simplex, indicating that a more equal distribution of political power and labor productivity will lead to more coalitions and hence a higher growth rate; (2) A minor change in the distribution of labor productivities (or interpreted as the adoption of a different technology) could result in a non-monotonic bifurcation in the coalition structure and in the macroeconomic performance; (3) The selfish ruling class in a non-democratic society might voluntarily relegate its political power to other social groups even without facing any threat of being overthrown, which might even achieve the highest growth rate incidentally. When the social groups are homogenous, our model suggests that inequality might arise endogenously even in the absence of any exogenous shocks as a result of the asymmetric Markov-Nash equilibrium.

One main contribution of this paper is that it represents an early attempt to rigorously model the endogenous formation of social coalition in the dynamic context of growth and inequality when the society is not necessarily democratic.[2] Most of the existing literature is concentrated in the exploration of the macroeconomic policies and performances under given social infrastructure, especially in the democratic society, see Persson and Tabellini (1999, 2003), Alesina and Rodrik(1994), Knack and Keefer (1995), Benabou(2000), etc.

[2] In the appendix, we analyze how the growth rate and inequality change dynamically with the social infrastructure when the political power distribution follows a Markov process.

A notable exception is Acemoglu and Johnson (2001), and especially the excellent monograph by Acemoglu and Robinson (2006). They study the policy making in the non-democracy and the consolidation of democracy. Nevertheless, their analysis is typically not through the prism of coalition formation. Moreover, they view the potential threat of coups or revolution as the main reason for the elite to unwillingly relegate their political powers to the "lower" social groups in the form of democratization, but our coalition analysis suggests that even without such threat the ruling class might still voluntarily extend some political rights to the other social groups in order to extract more surplus by facilitating more coalitions in the joint production, which might even cause the maximal growth. Our prediction seems quite consistent with China's recent rapid growth coupled with certain modest political reforms.

Another contribution is that the induced economic institutions and the induced technology adoption can be addressed simultaneously within our framework. This is because we can obtain the close-form value functions for each social group at any distributions of the political power and/or the labor productivity, which determine their induced preferences over different political institutions and their attitude towards any new technology. Parente and Prescott (2000) argue that the "barrier to riches" in many developing economies lies in the incumbent's economic monopoly power, which hampers the adoption of better technologies.[3] Acemoglu and Robinson (2006) disagree with their

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[3] In a finite period dynamic model, Krusell and Rios-Rull (1996) show that the incumbent workers might vote against adoption of a better technology for fear that their accumulated human capital would get obsolete.

"economic loser" story by proposing a "political loser" hypothesis: The incumbent refuse to adopt better technologies for fear that it would threaten their political power, otherwise they could simply allow for the adoption of the better technology and then tax it. Our model could help combine these two views as we examine the endogenous social infrastructure for any aggregate technology. In addition, we explicitly exploit the induced impact of endogenous social coalition on the long-run growth and consumption inequality.

The next section describes the model and characterizes the economic growth and consumption inequality under different time-invariant social coalition structures. Section 3 discusses how the social coalitions are endogenously formed. The bulk of our analysis will focus on the coalition formation with full commitment, although limited commitment is briefly discussed in relation with the coalition literature. Section 4 relates our model with more pertinent literature, where we also show how a few well-established models can be seen methodologically as the special cases of our model. The last section concludes.

2. Model Economy

Consider an economy with three infinitely lived representative agents, each of whom should be interpreted as an organized social group.[4] They jointly produce a good, which

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[4] There are several reasons to choose three players rather than any other number of agents. Firstly, it is the simplest case allowing for non-trivial coalition and social choice analysis. Secondly, if we interpret each agent as a social class, a society seems better described with a three-class model than a two-class model, as is well established in the political economy literature. One immediate application with a three-class model would be the determination of the pivotal class, or the median voter theorem in a democracy. If the redistribution policy is determined by the upper class, we call this an elite society, if by the middle-class, a democratic society, if the lower class, a leftist society. See, for example,

can be used for consumption or investment, with the following Cobb-Douglass production function

(1) $$y = Ak^{\alpha}\prod_{i=1}^{3} l_i^{\mu_i},$$

where A is the neutral technology coefficient, k is the total capital stock with the depreciation rate equal to 100%, l_i is the labor supply from the agent i. μ_i measures his labor productivity, which is allowed to be zero, so some agent might not be needed for the production. α is the contribution share of capital to the output, which is also allowed to be zero. However, we will take μ_i and α as positive unless explicitly specified otherwise. The property rights of the total output are not clearly defined, or clearly defined but insecurely protected. One case in point is the state-owned enterprises in the planned economies, where the on-job over-consumption by the managers and shirking are rampant.

Each agent has the following utility function

(2) $$\underset{\{c_t, l_t\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} \beta^t (\log c_t - \frac{1}{2} l_t^2),$$

where β is the time discount factor and c_t is the consumption at time t.

In each period there are five coalition possibilities in the joint production, namely, grand coalition, partial coalition (that is, two of them form a coalition, so there are three such possibilities), and no coalition. In contrast, there are only two possibilities in a

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Benabou (1996, 2000), Alesina and Rodrik(1994), Persson and Tabellini (2001), Acemoglu and Robinson (2006). etc. Thirdly, Acemoglu, Egorov and Sonin(2006) shows under a different setting why the three-player structure is most stable in the political coalition.

two-player game and seventeen possibilities in a four-player game. For simplicity, we exclude any multi-step coalition, that is, once a coalition is formed, these agents can't form any sub-coalitions in that period.[5]

Before analyzing the endogenous coalition decisions, let's first examine what happens if the coalition structure is time-invariant.

2.1 Permanent Full Coalition

When the agents fully cooperate with each other permanently, the social optimal problem is cast the following Bellman equation:

$$(3) \qquad V(k) = \underset{\{c_i, l_i\}}{Max}\{\sum_{i=1}^{3} a_i(\ln c_i - \frac{1}{2}l_i^2) + \beta V(Ak^{\alpha}\prod_i l_i^{\mu_i} - \sum_{i=1}^{3} c_i)\},$$

where the nonnegative parameter a_i measures the political power of agent i and $\sum_{i=1}^{3} a_i = 1$. For example, a_i might refer to the fraction of seats in the congress occupied by party i in a democracy with the proportional representation system. In a non-democracy, a_i can be interpreted as the political status of social group i as stipulated in the constitution. For instance, in the archetype communist society, the capitalist (bourgeoisie) has little, if any, political rights but the working class (proletarians) has a much larger political power. The ruling bureaucrats have an even larger power. For the time being, we assume these political power parameters are exogenous and constant. Later we will study what happens if they change over time and, in particular, how they

[5] If the multi-step coalition is allowed, there would be eight possibilities among a three-player model and sixty-two possibilities in a four–player model in each period! We will discuss the renegotiation-proof equilibrium later.

are determined endogenously.

In a democratic society, the permanent full coalition is interpreted as a consolidated democracy. Whereas in a dictatorship, it means that the ruler is powerful enough to maintain the control over the economy in an orderly manner. The consumption assignments could be interpreted as the outcome of certain implicit redistribution policies, which can be very general.[6]

We guess and verify $c_i = \lambda_i y$,[7] where $\lambda_i = a_i(1-\beta\alpha)$,

(4) $$c_i = a_i(1-\beta\alpha)y,$$

(5) $$l_i^2 = \frac{\mu_i}{a_i(1-\beta\alpha)},$$

(6) $$k' = \beta\alpha y,$$

(4)-(6) characterize the consumption, labor supply, and the investment, respectively. Equation (4) shows that each agent's consumption is a time-invariant fraction of the current total output. It also indicates that the equilibrium consumption inequality is solely determined by the distribution of the political power distribution. Equation (5) implies that an agent's labor supply increases in his labor productivity and decreases in his political power. Equation (6) states that a fixed proportion of the output will be invested in the next period.[8] By assuming $\alpha = 1$, as in the AK model, we can obtain the

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[6] One nice property is that we don't have to take a stand whether it should be a lump sum tax , linear tax, or nonlinear tax, etc.

[7] We can make this conjecture of the policy function because the utility function is log utility and the production function is of CES form. It can be proved that this simple form of policy function holds only when the risk aversion coefficient of the CES utility function is unity, and it holds even if the productivity A is a random variable. That's one important reason why we adopt this functional assumption.

[8] It can be easily proved that the investment share is always $\beta\alpha$ no matter how many agents there are in the economy.

sustainable economic growth with the following rate[9]

(7) $$g^f = \beta A \prod_{i=1}^{3} [\frac{\mu_i}{a_i(1-\beta)}]^{\frac{\mu_i}{2}} .$$

From (1), (2), (4), (5) and (6), we can obtain the value function of agent i:

(8) $$\begin{aligned} V^f{}_i(k) &= \frac{\alpha}{1-\beta\alpha} \ln k + \frac{1}{1-\beta} \{ \ln a_i (1-\beta\alpha) A \prod_{j=1}^{3} [\frac{\mu_j}{a_j(1-\beta\alpha)}]^{\frac{\mu_j}{2}} - \frac{1}{2} \frac{\mu_i}{a_i(1-\beta\alpha)} \\ &+ \frac{\alpha\beta}{1-\beta\alpha} \ln \beta\alpha A \prod_{j=1}^{3} [\frac{\mu_j}{a_j(1-\beta\alpha)}]^{\frac{\mu_j}{2}} \}, \quad \textit{for each } i \end{aligned}$$

2.2 Permanent No Coalition

When there is no cooperation permanently, it's a non-cooperative game. We use a nonnegative parameter p_i to denote the relative predation power of agent i, which is the fraction of the total output he is ensured to have after the fight of social conflict. One interpretation of p_i is agent i's military power[10]. Assume $\sum_{i=1}^{3} p_i = 1 - \eta$. The nonnegative parameter η measures the fraction of total output that is destroyed if the division of consumption cannot be settled peacefully and voluntarily.

The permanent-no-coalition scenario can be interpreted as a society in a complete political disorder, so the state is too weak.[11] In this "lawless" economy (to use Dixit's term), the agents first simultaneously determine their own labor supply at the current period and also announce the amount of output they want to have as their consumption, denoted as (m_1, m_2, m_3). Then the output is produced using the labor supply and the

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[9] We can easily obtain the steady state of the economy if we don't impose $\alpha = 1$.

[10] Grossman and Kim (1996) established a general equilibrium model to make this predation power (or defensive power) endogenous.

[11] Please see Acemoglu (2005) and Dixit (2001) for more discussions.

capital invested in the last period. The real consumption for any agent $i \in \{1,2,3\}$ is determined by the following rule, denoted as rule-1, which is commonly known.

$$c_i(m_1, m_2, m_3) = \begin{cases} m_i, \text{ if } & \sum_{i=1}^{3} m_i \le y, or\ i \notin I \\ \dfrac{p_i[(1-\eta)y - \sum_{j \in N/I} m_j]}{\sum_{j \in I} p_j}, \text{ if } & \sum_{i=1}^{3} m_i > y, and\ i \in I \end{cases},$$

where $I = \{i \in N : m_i > p_i y\}$.

That is, an agent will get what he claims if all the consumption claims are feasible or if his claim is no larger than what his predation power assures him of. If the claims are not feasible, then the brutal predation occurs, which destroys a fraction η of total output. Those who claim the amount within his predation power will still get what they claim, and the rest of the undestroyed output will be divided according to the relative predation powers among those agents who "greedily" claim more than what their predation power can ensure them. Whatever is left becomes the investment for the next period. It immediately implies that the investment will be zero whenever predation occurs and thus the output will be zero next period if $\alpha > 0$. Since the utility will be equal to minus infinity if the consumption is zero, the agent must take great caution to avoid predation when making their consumption announcement.

In the appendix, we discuss in more detail how to characterize the optimal strategies of each agent under various equilibrium definitions. Here we will focus on the Markov-Nash equilibrium. That is, each of the three agents will play a Cournot-game each period by choosing his own optimal labor supply and consumption while taking the other two agents' decisions as given.

Therefore, the problem faced by individual i is

$$V_i(k) = \underset{\{c_i, l_i\}}{Max}\{\ln c_i - \frac{1}{2}l_i^2 + \beta V_i(Ak^{\alpha}\prod_{j=1}^{3} l_j^{\mu_j} - \sum_{j=1}^{3} c_j)\} \quad (9)$$

We can guess and verify:

$$c_i = \frac{1-\beta\alpha}{3-2\beta\alpha} y, \quad (10)$$

$$l_i^2 = \frac{\mu_i}{1-\beta\alpha}. \quad (11)$$

Compared with full coalition, the labor supply decreases and the investment rate falls from the original $\beta\alpha$ to $\frac{\beta\alpha}{3-2\beta\alpha}$. Equation (10) says that the three agents will have the same consumption.[12] Notice that if each agent claims the amount in (10) and (11), then according to (rule-1), all the agents will have what they claim no matter what the predation power distribution is. We will show later that the consumption would in general depend on the predation power distribution if we use some different equilibrium concept, or if $\alpha = 0$, or if we change the utility function. The value function for any individual $i \in \{1, 2, 3\}$:

[12] We can easily extend this case to n-player game, where $\lambda = \frac{1-\beta\alpha}{1+(n-1)(1-\beta\alpha)}$ and $l_i^2 = \frac{\mu_i}{1-\beta\alpha}$.

(12)
$$V^n{}_i(k)=\frac{\alpha}{1-\beta\alpha}\ln k+\frac{1}{1-\beta}[\ln\frac{1-\beta\alpha}{3-2\beta\alpha}A\prod_{j=1}^{3}[\frac{\mu_j}{(1-\beta\alpha)}]^{\frac{\mu_j}{2}}-\frac{1}{2}\frac{\mu_i}{(1-\beta\alpha)}$$
$$+\frac{\alpha\beta}{1-\beta\alpha}\ln\frac{\beta\alpha}{3-2\beta\alpha}A\prod_{j=1}^{3}[\frac{\mu_j}{(1-\beta\alpha)}]^{\frac{\mu_j}{2}}]$$

The sustainable economic growth rate (assuming $\alpha=1$) is

(13)
$$g^n=\frac{\beta}{3-2\beta}A\prod_{i=1}^{3}[\frac{\mu_i}{(1-\beta)}]^{\frac{\mu_i}{2}}.$$

2.3 Permanent Partial Coalition

Without loss of generality, suppose agent 1 and agent 2 forms a permanent coalition while agent 3 doesn't cooperate. This can be reduced to a two-player game between the partial coalition and agent 3. First, an artificial planner for the partial coalition assigns the labor supply and consumption claims, (m_1, m_2), to agent 1 and agent 2. At the same time agent 3 supplies his labor and announces his consumption claim, m_3. Then the production is realized. The commonly known division rule, denoted as rule-2a, for the ultimate consumption of each period is the following:

For any $i\in Q\equiv\{1,2\}$,

$$c_i(m_1,m_2,m_3)=\begin{cases} m_i, & if \quad \sum_{j=1}^{3}m_j\le y \ or \ \sum_{j=1}^{2}m_j\le\sum_{j=1}^{2}p_j y \\ \dfrac{p_i[(1-\eta)y-m_3]}{\sum_{j=1}^{2}p_j}, & if \quad \sum_{j=1}^{3}m_j>y \ and \ m_3\le p_3 y \\ \dfrac{p_i(1-\eta)y}{\sum_{j=1}^{3}p_j}, & if \quad \sum_{j=1}^{2}m_j>\sum_{j=1}^{2}p_j y \ and \ m_3>p_3 y \end{cases}.$$

The real consumption for agent 3, c_3, is determined by the following rule-2b

$$c_3(m_1,m_2,m_3)=\begin{cases} m_3, & if \quad \sum_{i=1}^{3} m_i \le y, or\ m_3 \le p_3 y \\ (1-\eta)y-\sum_{j=1}^{2} m_j, & if \quad \sum_{i=1}^{3} m_i > y,\ \sum_{j=1}^{2} m_j \le \sum_{j=1}^{2} p_j y,\ and\ m_3 > p_3 y \\ \dfrac{p_3(1-\eta)y}{\sum_{j=1}^{3} p_j}, & if \quad \sum_{j=1}^{2} m_j > \sum_{j=1}^{2} p_j y,\ and\ m_3 > p_3 y \end{cases}.$$

This division rule is essentially similar to the rule for the permanent no coalition. Note that the partial coalition's predation power equals the sum of its members' predation power. Since we focus on the Markov equilibrium, the problem for agent 3 is

(14) $$V_3(k)=\underset{\{c_3,l_3\}}{Max}\{\ln c_3-\frac{1}{2}l_3^2+\beta V_3(Ak^{\alpha}\prod_i l_i^{\mu_i}-\sum_{i=1}^{3}c_i)\}.$$

We obtain

(15) $$c_3=\frac{1-\beta\alpha}{2-\beta\alpha}y,$$

(16) $$l_3^2=\frac{\mu_3}{1-\beta\alpha}.$$

The value function of agent 3 is

(17)

$$V^{1,2}{}_3(k)=\frac{\alpha}{1-\beta\alpha}\ln k+\frac{1}{1-\beta}\{\ln\frac{1-\beta\alpha}{2-\beta\alpha}A\prod_{i=1}^{2}[\frac{\mu_i(a_1+a_2)}{a_i(1-\beta\alpha)}]^{\frac{\mu_i}{2}}\cdot[\frac{\mu_3}{(1-\beta\alpha)}]^{\frac{\mu_3}{2}}-\frac{\mu_3}{2(1-\beta\alpha)}$$
$$+\frac{\alpha\beta}{1-\beta\alpha}\ln\frac{\alpha\beta}{2-\beta\alpha}A\prod_{i=1}^{2}[\frac{\mu_i(a_1+a_2)}{a_i(1-\beta\alpha)}]^{\frac{\mu_i}{2}}\cdot[\frac{\mu_3}{(1-\beta\alpha)}]^{\frac{\mu_3}{2}}\}$$

The problem facing the coalition of agent 1 and agent 2 is

(18) $$V^{1,2}(k)=\underset{\{c_i,l_i\}}{Max}\{\sum_{i=1}^{2}\frac{a_i}{a_1+a_2}(\ln c_i-\frac{1}{2}l_i^2)+\beta V^{1,2}(Ak^{\alpha}\prod_i l_i^{\mu_i}-\sum_{i=1}^{3}c_i)\}.$$

Similar to the full coalition case, the welfare distribution within a coalition is determined by the relative political power. We can obtain

(19) $$c_1=\frac{a_1}{a_1+a_2}\cdot\frac{1-\beta\alpha}{2-\beta\alpha}y,\ c_2=\frac{a_2}{a_1+a_2}\cdot\frac{1-\beta\alpha}{2-\beta\alpha}y,$$

(20) $$l_i^2=\frac{\mu_i(a_1+a_2)}{a_i(1-\beta\alpha)}\quad i=1,2.$$

The value function of the agent $j\in\{1,2\}$ is

(21)

$$V^{1,2}{}_j(k)=\frac{\alpha}{1-\beta\alpha}\ln k+\frac{1}{1-\beta}\{\ln\frac{a_j}{a_1+a_2}\frac{1-\beta\alpha}{2-\beta\alpha}A\prod_{i=1}^{2}[\frac{\mu_i(a_1+a_2)}{a_i(1-\beta\alpha)}]^{\frac{\mu_i}{2}}\cdot[\frac{\mu_3}{(1-\beta\alpha)}]^{\frac{\mu_3}{2}}-\frac{1}{2}\frac{\mu_j(a_1+a_2)}{a_j(1-\beta\alpha)}$$
$$+\frac{\alpha\beta}{1-\beta\alpha}\ln\frac{\beta\alpha}{2-\beta\alpha}A\prod_{i=1}^{2}[\frac{\mu_i(a_1+a_2)}{a_i(1-\beta\alpha)}]^{\frac{\mu_i}{2}}\cdot[\frac{\mu_3}{(1-\beta\alpha)}]^{\frac{\mu_3}{2}}\}$$

In fact, the partial coalition is exactly equivalent to a two-party non-cooperative game followed by a two-player cooperative game within one party. The consumption distribution among the three agents is $a_1:a_2:(a_1+a_2)$.

When $\alpha=1$, the economic growth rate is

(22) $$g_{1,2}{}^{pc}=\frac{\beta}{2-\beta}A\prod_{i=1}^{2}[\frac{\mu_i(a_1+a_2)}{a_i(1-\beta)}]^{\frac{\mu_i}{2}}[\frac{\mu_3}{(1-\beta)}]^{\frac{\mu_3}{2}}.$$

Similarly, we can obtain the corresponding results for the other two possibilities of partial coalition. By comparing (7), (13), and (22), we have

Lemma 1. $g^f>g^{pc}>g^n$ for any μ_i and a_i, $i=1,2,3$.

This lemma states that more coalition leads to a higher economic growth rate because more coalitions imply not only more labor supply from each agent but also a higher investment rate.[13] Therefore, the problem of how to maximize the economic growth is reduced to how to facilitate more coalitions.

Next we turn to the question how the social infrastructure is endogenously determined.

3. Endogenous Coalition

It is not a repeated game unless $\alpha = 0$. A full characterization of this general dynamic game with multidimensional continuous action space is complicated,[14] so we will mainly focus on the Markov Perfect equilibrium with full commitment, in which the coalition structure is time-invariant. In Subsection 3.2, we provide an informal short discussion on the alternative equilibrium concepts and the challenges of analyzing the equilibrium with limited commitment. The related literature on the coalition theory is also briefly discussed.

3.1 Full Commitment

At the beginning of the world, all the three agents need to choose simultaneously whether to cooperate or not. We allow for the pre-play cheap talk and assume that the economy will end up with the Pareto efficient equilibrium if there are multiple equilibria. The agent cannot choose with whom to cooperate. All the agents who have chosen to

[13] It seems quite consistent with China's experience described in the Introduction. The economic reform resulted in various forms of coalitions between the local bureaucrats and the rural workers or private businessmen, inducing a higher growth rate but more inequality. Before the reform, all these social groups worked passively and non-cooperatively and shirking was very rampant.

[14] To proceed in a standard way, we need to specify, for each agent, the payoffs for all the possible profiles of the strategies which are defined, at each date, as a mapping from the space of the history (the sequence of the past states and action profiles up to that date) and current state (capital stock) to the probability distribution on the relevant action space, which include the coalition decision, labor supply decision, as well as consumption decision. Please see the appendix for a more formal specification for this dynamic game.

cooperate form a coalition automatically. No coalition will be formed if two or three agents choose to not cooperate. By full commitment, we assume that once the coalition structure is realized, people can never renege on this coalition choice in all the future periods.[15] For future reference, we use ϖ^{f} to denote permanent full coalition, $\varpi^{i,j}$ to denote permanent partial coalition between agent i and agent j, $\forall i, j \in \{1,2,3\}, i \neq j$, and ϖ^{n} to denote no coalition forever. The original dynamic game can be transformed into its normal form with the following payoff matrices:[16]

[Insert Table 1-1 and Table 1-2 Here]

Recall we have already obtained the exact expression for each element in the above matrices. We focus on the pure-strategy equilibrium. We will show such equilibrium does exist by finding it.

The optimal strategy of each agent would, in principle, depend on the initial capital stock k_0 and the collection of the parameters $\{A, \alpha, \beta, \mu_1, \mu_2, \mu_3, a_1, a_2, a_3, p_1, p_2, p_3\}$. No result of generality can be obtained, so we will first examine what happens if all the agents are perfectly identical ex ante, and then explore several interesting and illuminating cases of heterogeneous agents.

3.1.1. Endogenous Coalition Among Identical Agents

When the three agents are perfectly identical, they have equal political power $a_1 = a_2 = a_3 = \frac{1}{3}$, equal predation power, and equal labor productivity. In particular, let's

[15] In the appendix, we provide an alternative description of this game in a cooperative game manner. It is equivalent to a static contracting problem combined with the social choice. We also show that it is trivially equivalent to a sequential contracting problem.

[16] In the appendix, we also explore how the coalition structure changes dynamically when the political power distribution follows an exogenous Markov process.

assume $\mu_1 = \mu_2 = \mu_3 = 1/3$.[17]

Proposition 1. *If the three agents are perfectly identical, then there exist two constants,* ζ_1 and ζ_2 such that $0 < \zeta_2 < \zeta_1 < 1$. *When* $\alpha\beta > \zeta_1$, *there are two pure strategy Nash equilibria: full coalition and no coalition; When* $\zeta_2 < \alpha\beta < \zeta_1$, *there are four pure strategy Nash equilibria: three partial coalitions and one no coalition; When* $\alpha\beta < \zeta_2$, *the only equilibrium outcome is no coalition. These equilibria are independent of the initial capital stock,* k_0, *and the neutral productivity coefficient,* A.

Proof. Compare the payoffs in Table1-1 and Table 1-2. ***Q.E.D***

This proposition shows that the capital share α and the time discount factor β are very important determinants for the equilibria. There tends to be more coalition when the agents become more patient (larger β) as the long-run gain from cooperation dominates the disutility from more labor supply in the coalition (see (5), (11), (16), (20)). A larger capital share α implies a higher marginal productivity of labor, which induces more labor supply and a higher investment rate, and hence more coalition. No coalition, although most inefficient in terms of growth rate by Lemma 1, is always an equilibrium due to the coordination failure as the " tragedy of commons".

In the appendix, we also show that full coalition Pareto-dominates no coalition when $\alpha\beta > \zeta_1$, and all the three partial coalitions Pareto-Dominate no coalition when $\zeta_2 < \alpha\beta < \zeta_1$. Pre-play communication would help select the more Pareto efficient Nash

[17] From now on, we will focus on the case when the three agents' labor productivities add up to one. This implies an increasing return to scale when $\alpha > 0$, which provides another incentive for the agents to form coalition. Even so, we will show that coalitions are not formed in many cases. If the technology demonstrates the constant or decreasing return to scale, we can proceed with the same analysis, but naturally, "no coalition" will be more likely.

equilibrium. So from now on, we will just focus on the Pareto efficient Nash equilibrium. Notice that whenever a partial-coalition equilibrium is reached (if $\zeta_2 < \alpha\beta < \zeta_1$), the agent excluded from the coalition is better off than any member within the partial coalition. One question is which one of the three partial equilibria will be indeed realized. Since we stay with the pure-strategy equilibrium, the only selection criterion is the sunspot argument: People's beliefs happen to be well coordinated on a particular equilibrium in a self-fulfilling manner.

Corollary 1.1. *Even if every agent is perfectly identical ex ante, all the information is public and complete, and there is no exogenous random shock to the preference or to the technology, it is still possible that the society ends up unequal if* $\zeta_2 < \alpha\beta < \zeta_1$.

Although very simple from a theoretic point of view, it points out one mechanism for endogenous inequality among ex ante identical agents in a complete information world. The existence of multiple equilibria thus results in the endogenous indeterminacy in the consumption distribution. This sunspot inequality results from the fact that the contract space is not large enough. Theoretically, an efficient and complete insurance market can alleviate all the strategic risks that each individual bears and hence maintain the equality of the society. However, such market or contract might be not available for practical reasons, especially in those less developed countries without secure property rights.

It is well known that the time discount factor is crucial for the multistage dynamic games, as discussed in various versions of the "Folk Theorem" for repeated games. What's relatively novel here is how the capital share in the production technology, α, can affect the equilibrium. The more capital intensive the technology, the more likely it is to generate more coalitions. If the technology is endogenously chosen by following the comparative advantage of the economy, then the more capital abundant economy tends to adopt the technology that has a larger α, hence the factor endowment will affect the equilibrium coalitional structure and hence affect the growth and consumption distribution. Although a more explicit model with endogenous technology choice is needed, Proposition 1 suggests a cross-sectional version of the Kuznets Curve:

Corollary 1.2. *Assume agents are perfectly identical. If the more capital abundant economy adopts the more capital-intensive technology, then from a cross-sectional point of view, the sufficiently poor economies (whose capital per capita is small) has the perfect consumption equality as it's in the no coalition equilibrium. Those mildly richer countries have more inequality as they are in the partial coalition equilibrium. Those sufficiently rich countries will also enjoy the perfect equality because they are in the full coalition equilibrium.*

3.1.2 Endogenous Coalition Among Heterogeneous Agents

There is only one way to be homogeneous, but there are numerous ways to be heterogeneous. We can therefore only experiment with several simple examples that seem illustrative.

Example One: Meritocracy

Consider a meritocracy where the three agents' political powers are proportional to their labor productivities. That is, $\mu_i = a_i$, $i = 1,2,3$. These agents are perfectly identical in all the other aspects. Proposition 1 is a special case of this example. It's straightforward to show that a necessary condition to have full coalition is $\alpha\beta \geq \zeta_1 \approx 0.55671$. For computational purpose, we arbitrarily set $\alpha\beta = 0.9$ throughout the numerical exercises. The result can be best illustrated on the following simplex:

[Insert Figure 1 Here]

The three rhombi A, C, and E correspond to "No Coalition". The middle equilateral G is "Full Coalition". The three trapezoids B, D, F are "Partial Coalitions". In particular, in B, agents 1 and 3 are both sufficiently productive and form a partial coalition. In D, players 1 and 2 form a partial coalition while in F, 2 and 3 are partners. So the consumption ratio (inequality) is $a_1 : a_2 : a_3$ (equal to $\mu_1 : \mu_2 : \mu_3$) in the equilateral G. It is 1:1:1 in area A, C, and E. It's $a_1 : a_2 : a_1 + a_2$ in area D, $a_2 + a_3 : a_2 : a_3$ in area F, and $a_1 : a_1 + a_3 : a_3$ in area B. We have also obtained the growth rate for each region.

From the payoff matrices, we can show that full coalition requires each agent's political power (or labor productivity) be sufficiently large, or more specifically, $\mu_i \geq \xi \approx 0.26658, \forall i = 1,2,3$. Moreover, suppose the distribution of (μ_1, μ_2, μ_3) is randomly drawn from a uniform distribution on the simplex $\Delta = \{(\mu_1, \mu_2, \mu_3) \in \mathbb{R}_+^3 : \sum_{i=1}^{3} \mu_i = 1\}$, then the likelihood ratio of full coalition vs. partial coalition vs. no coalition is $\frac{(1-3\xi)^2}{2} : (3\xi - 7.5\xi^2) : 3\xi^2$. Hence it suggests that a more equal distribution of labor productivity and political power will in general cause more coalition and a higher growth rate.

Example Two: Heterogeneous Productivities

In this example, the three agents are perfectly identical except for their labor productivities. To simplify, we further assume that agent 1 and agent 2 have equal productivities, $\mu_1 = \mu_2$, and all the three agents' labor productivities add up to unity. For this particular parametric example, the coalition structure changes non-monotonically when the productivities change, as shown in the following table:

[Insert Table 2 Here]

It's interesting that partial coalition can be achieved only when the two agents with the same productivities are extremely unproductively or sufficiently productive. When they are both extremely unproductive, they want to form the coalition in order to grab more consumption from the most productive agent, namely, agent 3. This free-ride motive dominates the increased disutility from more labor supply as compared with the no coalition case. When agents 1 and 2 are both sufficiently productive and agent 3 is extremely unproductive, agent 3 chooses to stay out of the coalition to free-ride on their relatively high productivity, and at the same time agents 1 and 2 want to form the partial coalition due to the complementarity of their labors in the production. Table 2 also shows that the coalition structure changes from no coalition to full coalition once the labor productivity of two identically productive agents exceeds 0.08 from below. This causes a qualitatively huge improvement in the macroeconomic performance, because the social gain is much more than the two individuals' increases in the labor productivity, but more importantly, it induces a dramatic structural change in the social infrastructure, which actually eliminates the free-ride problem in the society (hence induce higher investment rate and labor supply). However, if the two equally productive agents are extremely unproductive (slightly below 0.01837), then a minor increase in the two agents' productivities would dissolve their partial coalition and return the economy to the worst institution with permanent no coalition.

This example suggests vividly that a small difference in the labor productivity distribution might imply a dramatic difference in the social infrastructure and macroeconomic performance. Moreover, we can easily derive the value function for each social group at any such productivity distribution, which allows us to pin down which kind of technology shall be finally adopted so long as we specify the mechanism of the aggregate decision. For example, it might be decided by the ruling social group if it's dictatorship, or it might be decided through voting by all the groups in the democracy.

Example Three: Heterogeneous Political Power

In example one, the political power and the productivity are tied together. Now we disentangle them by assuming that the agents are only different in the political power while holding their productivities perfectly equal, that is, $\mu_1 = \mu_2 = \mu_3 = 1/3$. The endogenous coalition behaviors can be illustrated in Figure 2.

[Insert Figure 2 Here]

The triangle G refers to full coalition. Areas A, C, E represent No Coalition. Trapezoid B is the partial coalition between agents 1 and 3. Trapezoid D is the partial coalition between agents 1 and 2. Trapezoid F is the partial coalition between agents 2 and 3. There are six dashed lines *a, b, c, d, e, f,* which overlaps with the six unparalleled sides of the three trapezoids. For example, part of line *a* overlaps with the right side of the trapezoid B. Along this side, agent 3 is indifferent between no coalition and the partial coalition with agent 1. On the common border of trapezoid B and area A, agent 1 is indifferent between no coalition and partial coalition with 3. Moreover, agent 2 obtains the same value for any point on the two unparalleled sides. On the common border of trapezoid B and triangle G, agent 2 is indifferent between full coalition and partial coalition by 1 and 3. However, the values for agent 1 and agent 3 will jump if agent 2 shifts his coalition choice on this border. Furthermore, on the bottom side of triangle G, agent 2 achieves the lowest value at the midpoint, but the value is monotonically increasing as it moves towards the two endpoints. On the right apex of the triangle G, for example, both agents 2 and 3 are on the margin of full coalition, once either of them chooses to not cooperate, the full coalition will immediately collapse to no coalition because each of them have too small a political power relative to that of agent 1.Using symmetry, we can derive similar properties for the trapezoids D and F.

This figure delivers a similar message as in the first example. Namely, a more equal distribution of the political power would imply more coalition among the agents and hence a higher economic growth rate. This example also suggests that people with similar political powers are more likely to form a coalition as they expect to get a more equal share within the coalition.

Endogenous Political Power by Nash Implementation

All the above examples take the political power distribution as exogenous. Now suppose one of the three agents, say agent 1, represents the elite class, who can decide the political power distribution over the simplex $\Upsilon = \{(a_1, a_2, a_3) \in \mathbb{R}_+^3 : \sum_{i=1}^{3} a_i = 1\}$ right before the coalition decisions are made at the beginning of the world. The allocation of the political power can be interpreted as the establishment of the political institutions that govern the political powers.[18] What will be the optimal choice for agent 1?

The optimization algorithm is simple. We can obtain the maximized values for agent 1 when he Nash implements each of the five possible coalition structures by manipulating the political power distributions. Then agent 1 chooses the one that gives him the largest utility. Recall that the coalition decision is independent of the capital stock k and the neutral technology coefficient A, so the political power allocation is also independent of them.

Within the subset of Υ that generates full coalition, denoted as $A(\varpi^f) \subseteq \Upsilon$, agent 1 can maximize his value (8) by setting $a_1 = 1 - 2\varphi$ and $a_i = \varphi, \forall i = 2,3$, which corresponds to the right end point of the triangle G, where the consumption distribution is $1 - 2\varphi : \varphi : \varphi$. His value is denoted as $\tilde{V}_1(\varpi^f)$, given by (8).

If agent 1 Nash implements the partial coalition between agent 2 and agent 3, denoted as $\varpi^{2,3}$, he maximizes his value function by choosing the political power

[18]For more discussions about the *de facto* and *de jure* institutions, please see Acemoglu et al (2005, 2006).

distribution from the subset $A(\varpi^{2,3}) \subseteq \Upsilon$, that is, the trapezoid F. Denote his maximized value by $\tilde{V}_1(\varpi^{2,3})$. It can be shown that the maximized value is achieved for any points on the two unparalleled sides of the trapezoid F. It turns out that $\tilde{V}_1(\varpi^{f}) > \tilde{V}_1(\varpi^{2,3})$.

If agent 1 Nash implements the partial coalition between 1 and 2, denoted by $\varpi^{1,2}$, he can maximize his value by choosing any point on the common border of area D and area C. But the value is smaller than $\tilde{V}_1(\varpi^{f})$. By symmetry, we know that agent 1 also prefers full coalition to $\varpi^{1,3}$. We can further verify that agent 1 prefers full coalition to no coalition, denoted by ϖ^{n}. That is, $\tilde{V}_1(\varpi^{f}) > \tilde{V}_1(\varpi^{n})$.

In summary, for this particular set of the parameter values, agent 1 will choose to implement the full coalition by setting $a_1 = 1 - 2\varphi$ and $a_i = \varphi, \forall i = 2,3$, which turns out to be growth maximizing as well. There are two other growth-maximizing distributions, in each of which two agents have the political powers equal to φ. Obviously, whoever has the political power of assigning political powers will be strictly better off.

This example might be suggestive when we try to understand why China's central government, although authoritatively powerful, has been smoothly delegating more economic autonomy to the local governments and granting more political rights to the class of the private entrepreneurs. At the same time, China's economic growth rate has been remarkably high, close to an annual average 9% for more than two decades.

3.2 Informal Discussion on Limited Commitment

With limited commitment, the coalition structure might change over time. In fact, Acemoglu and Robinson (2006) argue that the concern of limited commitment necessitates institution buildings in the political negotiations between different social groups. The formal specification of the dynamic game is relegated to the appendix. It's natural to start with the sub-game perfect equilibrium. Obviously, it is a sub-game perfect Nash equilibrium that all the agents choose to not cooperate each period and claim the whole output as their consumption (with any amount of labor supply), because it will result in zero investment and zero output (if $\alpha = 0$), hence minus infinity utility. Then we can use this sub-game perfect equilibrium as the punishment rule to support any strategy profile that generates finite utility as a sub-game perfect Nash Equilibrium. In particular, it could support the permanent full coalition as an equilibrium, which we know is also Pareto optimal. This result has the similar flavor of the various versions of the Folk Theorem in the repeated game.

However, the punishment rule described above is not coalition-proof since all the three agents can jointly deviate to another strategy profile such that all of them are strictly better off. Ideally we wish to find a set of Pareto-efficient strategy profiles as the punishment strategy to support a set of Pareto-efficient strategy equilibria so that we can obtain the renegotiation-proof equilibrium. It is difficult because the Pareto efficient set is not compact mainly due to the logarithm utility function.

Another challenge comes from the fact that each player has a multidimensional and continuous action space. By contrast, most of the multilateral dynamic coalition analyses in the literature are focused on the one-dimension and finite or at most countable action spaces, see, for example, Abreu. Pearce, and E. Stachetti (1993), Konishi and Ray (2003).

Some others focus on the equilibrium size or structure of the coalitions when the number of players is any arbitrary $n \geq 3$. Such examples include the public goods or joint production analysis by Ray and Vohra (1997, 2001) and Ray (2006), and the political coalitions analysis by Acemoglu, Ergorov, and Sonin (2006). Traditionally, the coalition analysis involves the Sharpley value or cores with transferable or non-transferable utilities. Although all very inspiring, they mainly deal with the static economies with no accumulative assets or no production as in our model. Moreover, the action space in these models has a lower dimension.

As such, the task of providing a thorough analysis on the equilibrium with limited commitment appears very challenging, left as a top priority on our future research agenda. Let's end this discussion by mentioning that among the Markov-Perfect Nash Equilibria with full commitment that we have characterized in subsection 3.1, some might be already coalition-proof. For example, when all the three agents are perfectly identical, we show in the appendix that if $\alpha\beta > \zeta_1$, the full coalition equilibrium is most preferred by all the agents for any level of capital stock and hence must be coalition-proof in all the subgames. In other words, it is also the equilibrium with limited commitment.

4. Related Literature

Technically, our model is essentially an extension of the two-agent model by Benhabib and Rustichini (1996) (henceforth, BR model). The main differences between us are two folds.

First, the main purposes are different. The BR model mainly tries to explain how the insecure property right results in low investment rate and hence low growth rate. By contrast, our main point is to explore the endogenous coalition formation among the identical or heterogeneous social groups as well as the macroeconomic implications implied by these social infrastructures. As such, the bulk of our analysis is to show how the different social infrastructures come into being under different distributions of the political powers and/or the productivities. In addition, we also show how the political power distribution can be endogenously determined by a ruling class aiming to exploit the largest surplus. The three-agent setting also shows how inequality might arise from ex ante identical agents as the asymmetric equilibrium. These are all absent in the two-agent BR model. To be fair, the analysis of BR model entertains the possibility that the inequality might change dynamically, while our analysis with full commitment implies a time-invariant growth rate and consumption inequality. Nevertheless, we could obtain the more interesting dynamics in the coalition structure, growth rate, and inequality by introducing a Markov process for the political power distribution, as is shown in the appendix.

Second, from a methodological point of view, our model generalizes the BR model along several directions. One is that the increase in the number of players allows for the possibility of coalition analysis and the intermediate states of strategic cooperation. Second, we have introduced the explicit predation power distribution. Although it's not fully exploited in our analysis, it might potentially change the reservation values of different players and hence their equilibrium outcomes, especially if the utility function is finite at zero consumption or if capital share, α, is zero. It also admits further discussions on the endogenous property right.[19] All these features will allow for much more strategic possibilities and richer macroeconomic implications than the BR model.

Interestingly, our model can be seen as a generalized version of several well-established models in the literature. When $N = \{1\}$, it's nothing but the standard Ramsey growth model. When $\alpha = \mu_1 = \mu_2 = \mu_3 = 0$, it becomes a pure endowment economy. In particular, if $N = \{1, 2\}$, $\{a_1, a_2\}$ are not specified, and both A and $\{p_1, p_2\}$ follow a two-state Markov process, then it becomes exactly the same as the model of consumption insurance with limited commitment, as is studied by Ligon, Thomas and Worrall (2002, RES). When A remains fixed and $\{p_1, p_2\}$ follows a 0-1 Markov process, it degenerates into the political compromise model by Dixit, Grossman, and Gul (2000, JPE). If $\{p_1, p_2\}$ follows an *i.i.d* process, it becomes the risk-sharing model by Kocherlakota (1996, JME). Genicot and Ray (2003) study the group risk sharing by mainly focus on the number of the agents that will participate in the risk-sharing contracts with limited commitment.

5. Conclusion

We believe that the macroeconomic performance of different developing counties can be understood more deeply if we know more about the underlying social infrastructure. Thus this paper presents a qualitative exploration on how the coalition is endogenously formed among different social groups and how the resulted social infrastructure affects the

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19 See Acemoglu and Johnson(2005), Grossman and Kim (1996), etc.

long-run economic growth and consumption inequality. We emphasize the role of the political power distribution as well as the labor productivity distribution in determining the social infrastructure.

Among other things, we provide a numerical example to show that the ruling class, even when facing no risk of being overthrown, might voluntarily relegate the political rights to the other social groups in order to pursue its own interest, which even incidentally maximizes the economic growth rate. We also show how a minute alteration in the distribution of productivities might cause drastic variation in the social structure, which can potentially determine the attitudes of different social groups towards the adoption of any new technologies. In addition, we demonstrate that several well-established models in the literature all can be seen as the special cases of our model.

When applied to China, our model predicts that more coalitions imply a higher growth rate and that partial coalitions cause more consumption inequality, which is well aligned with the anecdotes we described in the introduction. Moreover, the political power redistribution reflected by China's constitution modifications, together with China's social coalition structure and macroeconomic performance, seems to be consistently and coherently explained within our framework for heterogeneous groups. A more detailed quantitative evaluation of our model for China and other countries is very desirable but could be challenging due to the stark nature of our model, hence it deserves a separate paper for a full-fledged analysis. The bottom line of this paper, nevertheless, is trying to convince the readers that the qualitative thought experiment conducted here might help us gain some non-trivial new insights in the relationship between the endogenous social infrastructure and the macroeconomic performance, especially for a developing economy like China.

Appendix

A. A More Formal Specification of the General Dynamic Game

1. Notations:

Define the time set $T \equiv \{0,1,2,...\}$. We have an extensive form game $\Gamma^e = (N, \{H^t\}_{t\in T}, \underset{\substack{i\in N \\ t\in T}}{\times} U_{i,t})$, where $N = \{1,2,3\}$ is the set of players, H^t is the set of all the histories up to time t, $U_{i,t}$ denotes the total present discounted value function of agent i at time t, $\forall t \in T$. I will explain in details how H^t and $U_{i,t}$ are defined very soon. At each date t, all the three agents will play a simultaneous game, which shall be described in details soon. Let's define this game as the simplest non-cooperative game.

Define $S_{i,t} = \Psi_{i,t} \times L_{i,t} \times M_{i,t}$, where $\Psi_{i,t} = \{1,\ 0\}$ is the space of coalition decisions by agent i at time t, 1 denotes "cooperate" and 0 denotes "not cooperate". **The player can't choose his partner**. When the player chooses 1, he **must** form a "coalition" with anyone who also chooses 1 at that period. $L_{i,t} = \mathbb{R}_+$ is the space of the labor supply by agent i at time t; $M_{i,t} = \mathbb{R}_+$ is the space of the "attempted" consumption by agent i at time t. Let $K_t \subseteq \mathbb{R}_+$ denote the space of the capital stock at time t. The world starts at time 0 with an exogenous initial capital stock equal to k_0.

A history up to time t is denoted by $h^t = (k_0, (k_1, s_0), (k_2, s_1), ,..., (k_t, s_{t-1}), (k_{t+1}, s_t))$, where $s_t \in S_t \equiv \underset{i\in N}{\times} S_{i,t}$ and $k_t \in K_t$. So the set of the histories up to date t is $H^t = (K_0, K_1 \times S_0, K_2 \times S_1, K_3 \times S_2, ..., K_{t+1} \times S_t)$. We will use $C_{i,t} \in \mathbb{R}_+$ to denote the space of the real consumption of agent i at time t. Let $c_{i,t}$ denote an element of this set.

Define, for each agent $i \in N$ at any time $\tau \in T$, the total sum of the discounted utility after that period:

$$U_{i,\tau}(\{l_{i,t}, c_{i,t}\}_{t=\tau}^{\infty}) = \sum_{t=\tau}^{\infty} \beta^{t-\tau}(\ln c_{i,t} - \frac{l_{i,t}^2}{2}), \tag{A-1}$$

Which is equal to minus infinity if $c_{i,t} = 0, \ \ for \ \ some \ t \geq \tau$.

2. Strategies

We should think that the stage game at time t has two steps. The first step is to make the coalition decision. Let's define the pure strategy of agent i's coalition decision at time t as the mapping:

$$\psi^i{}_t : H^{t-1} \rightarrow \Psi_{i,t}. \tag{A-2}$$

The coalition formation at time t is then realized, denoted by function

$$g : \Psi_{1,t} \times \Psi_{2,t} \times \Psi_{3,t} \rightarrow \Omega,$$

Where $\Omega = \{\varpi^f, \varpi^{1,2}, \varpi^{2,3}, \varpi^{1,3}, \varpi^n\}$, the five elements of which refer to the full coalition, the partial coalition between agent 1 and agent 2, the partial coalition between agent 2 and agent 3, the partial coalition between agent 1 and agent 3, and no coalition, respectively. This coalition structure is only binding at that period. The coalition formation function is defined as:

$g(1,1,1) = \varpi^f$;

$g(0,1,1) = \varpi^{2,3}; g(1,0,1) = \varpi^{1,3}; g(1,1,0) = \varpi^{1,2}$;

$g(0,0,1) = g(0,1,0) = g(0,0,1) = g(0,0,0) = \varpi^n$

The second step is to define each agents' production and consumption strategies after observing the realized coalition structure AND each individual's coalition decision at that period t.

Possibility 1: ϖ^f is realized at period t.

If full coalition, ϖ^f, is realized at time t, then for each agent $i \in N$, the real consumption $c_{i,t}$ and the real labor supply $l_{i,t}$ are not freely chosen by those individuals but rather assigned by a fictitious planner for the full coalition, and all the agents have to accept the assigned values. The abstract assignment function is a mapping:

$$D_t(\omega^f) : H^{t-1} \rightarrow (\underset{i \in N}{\times} C_{i,t}, \underset{i \in N}{\times} L_{i,t}). \tag{A-3}$$

This assignment function is determined by maximizing a weighted sum of the three agents' total welfare from that period on. The welfare weights, denoted by $\{a_1, a_2, a_3\}$, are interpreted as the *political* power structure of the three agents. For the time being, suppose they are exogenous and constant over time, later we will discuss what happens if the power distribution is endogenous or follows an exogenous Markov process. Denote the whole space of *political* power structure by

$\Upsilon = \{(a_1, a_2, a_3) \in \mathbb{R}^3_+ : \sum_{i \in N} a_i = 1\}$. So the fictitious planner solves the following problem at time t based on his information $H^{t-1} \times \Psi_{1,t} \times \Psi_{2,t} \times \Psi_{3,t}$:

$$\max_{(\times_{i \in N} c_{i,t}, \times_{i \in N} l_{i,t})} \sum_{i \in N} a_i U_{i,t} . \tag{A-4}$$

Obviously, in order to solve this problem, the fictitious planner needs to have an expectation about what happens in all the future periods. For example, it's possible that in some future time point no coalition is formed so the real consumption and labor supply at that period are not directly assigned by the social planner (See below for more details about what will happen in that case). We impose the rational expectation (perfect foresight) assumption for this fictitious planner whenever he is called on to make assignments.

Possibility 2: No coalition, ϖ^n , is realized at time t.

Each agent $i \in N$ needs to simultaneously claim his "attempted" amount of consumption, $m_{i,t} \in M_{i,t}$, and decides his labor supply, $l_{i,t} \in L_{i,t}$. That is, for $\forall i \in N$, his pure strategy of the consumption claim and the labor supply is a mapping:

$$\kappa_{i,t} : H^{t-1} \times \Psi_{1,t} \times \Psi_{2,t} \times \Psi_{3,t} \rightarrow M_{i,t} \times L_{i,t} . \tag{A-5}$$

Then output is produced using the inherited capital stock and the agents' labor supplies:

$$y_t = A k_t^{\alpha} \prod_{i=1}^{3} l^{\mu_i}{}_{i,t}$$

The real consumption for each agent $i \in N$ at time t, $c_{i,t}$, is determined by the commonly known rule (rule-1) in the main text (suppressing the time t subscript), where $I = \{i \in N : m_i > p_i y\}$ and the non-negative parameter p_i denotes the predation power of the agent i.

Let's now simply assume that $p_1 = p_2 = p_3 = small\ \ positive\ number$[20].

Possibility 3: A partial coalition, for example, $\varpi^{1,2}$, is realized.

The agent who is out of the coalition, agent 3 in this case, needs to decide his pure strategy of the consumption claim and the labor supply:

[20] The reason to introduce p is to allow for the possibility of asymmetric distribution of the labor supply and consumption decisions under no coalition, which seems to be more realistic.

$$\kappa_{3,t}(\varpi^{1,2}): H^{t-1} \rightarrow M_{3,t} \times L_{3,t} \ . \tag{A-6}$$

For each agent $i \in Q \equiv \{1,2\}$, the real labor supply $l_{i,t}$ is assigned by the fictitious planner for that partial coalition and both members of that partial coalition have to accept the assigned values. Like the full coalition, the fictitious planner for the partial coalition decides how to set the labor and consumption assignments (that is, split the real consumption for agents1 and 2 at time t, $(m_{1,t}, m_{2,t})$) by solving the following problem at time t based on the information $H^{t-1} \times \Psi_{1,t} \times \Psi_{2,t} \times \Psi_{3,t}$:

$$\max_{(\times_{i \in Q} M_{i,t}, \times_{i \in Q} L_{i,t})} \sum_{i \in Q} \frac{a_i}{\sum_{j \in Q} a_j} U_{i,t} \ . \tag{A-7}$$

Similarly, we also need the rational expectation or even perfect foresight assumption for this planner. Let's denote this assignment function as

$$D_t(\omega^{1,2}): H^{t-1} \rightarrow (\times_{i \in Q} M_{i,t}, \times_{i \in Q} L_{i,t}) \tag{A-8}$$

The output is produced using all the three agents' labor supplies and the capital. The fictitious planner for the partial coalition claims a consumption pair for that partial coalition $(m_{1,t}, m_{2,t}) \in M_1 \times M_2$.

Recall agent 3 claimed $m_{3,t}$.

The real consumption in the partial coalition, $c_{i,t}$, for any $i \in Q \equiv \{1,2\}$ and the real consumption for agent 3, $c_{3,t}$ are determined by (rule-2a) and (rule-2b) in the main text that are commonly known.

Symmetrically, we can also describe the consumption division rule for the other two partial coalitions in a similar manner.

The implied evolution of the capital stock is

$$k_{t+1} = A k_t^{\alpha} \prod_{i=1}^{3} l_{i,t}^{\mu_i} - \sum_{i=1}^{3} c_{i,t} \ . \tag{A-9}$$

Note that the feasibility constraint has been taken care of by the division rule for the real consumption.

Just for the ease of exposition, we define the game such that after observing the realized coalition structure at each period, no matter what it is, all the agents can always "announce" their own "attempted" labor supply and the "attempted" consumption. Of course, when full coalition or any of the three partial coalitions is realized, some agents will have to accept their assigned values for the labor supply and consumption claim, in which cases their own claims can't affect the payoff and are simply ignored.

3. Equilibrium Concepts

First note that this dynamic game is not a repeated game unless $\alpha = 0$.

Definition 1: *For the game* $\Gamma^e = (N, \{H^t\}_{t\in T}, \underset{\substack{i\in N\\ t\in T}}{\times} U_{i,t})$ *, a pure-strategy sub-game perfect equilibrium is a collection of the pure behavioral strategies defined in (A-2) through (A-8) such that for each agent* $i \in N$ *, at each time* $\tau \in T$ *, it must be true that:*

$$U_{i,\tau}(\{s^*_{i,t}\}^\infty_{t=\tau}, \{s^*_{-i,t}\}^\infty_{t=\tau}) \geq U_{i,\tau}(\{s_{i,t}\}^\infty_{t=\tau}, \{s^*_{-i,t}\}^\infty_{t=\tau}), \forall \{s_{i,t}\}^\infty_{t=\tau} \in \{S_{i,t}\}^\infty_{t=\tau}$$

And for any coalition result $g(\psi^*_{1,t}, \psi^*_{2,t}, \psi^*_{3,t})$,

$$U_{i,\tau}(\{l^*_{i,t}, m^*_{i,t}, \psi^*_{i,t+1}\}^\infty_{t=\tau}, \{l^*_{-i,t}, m^*_{-i,t}, \psi^*_{-i,t+1}\}^\infty_{t=\tau}) \geq U_{i,\tau}(\{l_{i,t}, m_{i,t}, \psi_{i,t+1}\}^\infty_{t=\tau}, \{l^*_{-i,t}, m^*_{-i,t}, \psi^*_{-i,t+1}\}^\infty_{t=\tau}),$$
$$\forall \{l_{i,t}, m_{i,t}, \psi_{i,t+1}\}^\infty_{t=\tau} \in L_{i,t} \times M_{i,t} \times \{S_{i,t+1}\}^\infty_{t=\tau}$$

This definition makes sure that it is a Nash equilibrium in all the possible sub-games, so it has to be sub-game perfect.

Note that there always exist a continuum of trivial *pure-strategy sub-game perfect* equilibria, in which all the three agents choose not to cooperate and set their labor supply equal to zero (or claimed consumption equal to the total output) all the time[21]. This implies that the output (or investment rate) is zero at that period and the total utility of each agent is negative infinity because of the log utility on consumption. No one can become strictly better off by making any unilateral change in his strategies.

Notice that any production and consumption choices under full coalition or any partial coalition are also feasible under no coalition. The only difference is that the players within a coalition lose their control power for that period, or in other words, coalition serves as nothing but a temporary commitment scheme[22].

A sharp characterization for this general dynamic game, especially the cases of no coalition or partial coalition, might be very hard, so we will first focus on the following Markov pure-strategy equilibrium. That is, instead of looking at the strategies based on the whole history as defined in (A-2) through (A-7), we look at the Markov pure strategies defined as

$$\tilde{\psi}^i_t : K_t \times S_{t-1} \to \Psi_{i,t}. \quad \text{(A-2')}$$

$$\tilde{D}_t(\omega^f) : K_t \times S_{t-1} \to (\underset{i\in N}{\times} C_{i,t}, \underset{i\in N}{\times} L_{i,t}). \quad \text{(A-3')}$$

$$\tilde{\kappa}_{i,t} : K_t \times S_{t-1} \times \Psi_{1,t} \times \Psi_{2,t} \times \Psi_{3,t} \to M_{i,t} \times L_{i,t}. \quad \text{(A-5')}$$

---

[21] We can assume that the agent will be still alive even after having zero consumption at certain period.

[22] If we modify this game such that the players cannot make any coalition choices at any period and hence all the production and consumption decisions are made under no coalition, then there is no commitment at all. The game becomes simpler but the general analysis might be still very difficult. If, however, we focus on the Markovian equilibrium and ASSUME that the players are playing a Cournot-type game under no coalition, then we will need this pre-commitment coalition decision to simplify our analysis of partial coalition and full coalition. Hence the whole analysis is based on non-cooperative game instead of the cooperative game. Ideally, we should determine the bargaining power of the three players endogenously when they are forming coalitions in a more standard way. I have not figured it out thoroughly and will discuss this later.

$$\tilde{\kappa}_{3,t}(\varpi^{1,2}): K_t \times S_{t-1} \to M_{3,t} \times L_{3,t}\,. \quad \text{(A-6')}$$

$$\tilde{D}_t(\omega^{1,2}): K_t \times S_{t-1} \to (\underset{i\in Q}{\times} M_{i,t}, \underset{i\in Q}{\times} L_{i,t}) \quad \text{(A-8')}$$

(A-4') and (A-7') are different from (A-4) and (A-7) only in that the former are determined under the information $K_t \times S_{t-1} \times \Psi_{1,t} \times \Psi_{2,t} \times \Psi_{3,t}$ instead of the whole history.

Definition 2: *For the game* $\Gamma^e = (N, \{H^t\}_{t\in T}, \underset{\substack{i\in N\\ t\in T}}{\times} U_{i,t})$*, a Markov pure-strategy sub-game perfect equilibrium is a collection of the behavioral strategies defined as (A-2') through (A-8') such that for each agent* $i \in N$*, at each time* $\tau \in T \equiv \{0,1,2,...\}$*, it must be true that:*

$$U_{i,\tau}(\{s^*_{i,t}\}^{\infty}_{t=\tau}, \{s^*_{-i,t}\}^{\infty}_{t=\tau}) \geq U_{i,\tau}(\{s_{i,t}\}^{\infty}_{t=\tau}, \{s^*_{-i,t}\}^{\infty}_{t=\tau}), \forall \{s_{i,t}\}^{\infty}_{t=\tau} \in \{S_{i,t}\}^{\infty}_{t=\tau}\,,$$ *and for any coalition result* $g(\psi^*_{1,t}, \psi^*_{2,t}, \psi^*_{3,t})$,

$$U_{i,\tau}(\{l^*_{i,t}, m^*_{i,t}, \psi^*_{i,t+1}\}^{\infty}_{t=\tau}, \{l^*_{-i,t}, m^*_{-i,t}, \psi^*_{-i,t+1}\}^{\infty}_{t=\tau}) \geq U_{i,\tau}(\{l_{i,t}, m_{i,t}, \psi_{i,t+1}\}^{\infty}_{t=\tau}, \{l^*_{-i,t}, m^*_{-i,t}, \psi^*_{-i,t+1}\}^{\infty}_{t=\tau}),$$
$$\forall \{l_{i,t}, m_{i,t}, \psi_{i,t+1}\}^{\infty}_{t=\tau} \in L_{i,t} \times M_{i,t} \times \{S_{i,t+1}\}^{\infty}_{t=\tau}$$

Before we try to provide a sufficient characterization of this equilibrium, let's first look at the Markov pure-strategy subgame perfect equilibrium with the additional restriction that the coalition structure is invariant over time. That is, we focus on the set of the *coalition-invariant strategies* defined as follows:

A Markov pure-strategy sub-game perfect equilibrium $\sigma_{i,t}: K_t \times S_{t-1} \to S_{i,t}$ *for each agent* $i \in N$ *at each time* $t \in T$ *as defined in (A-11) is called coalition-invariant if the value of* g *as defined in (A-2) remains constant over time.*

Definition 3: *For the game* $\Gamma^e = (N, \{H^t\}_{t\in T}, \underset{\substack{i\in N\\ t\in T}}{\times} U_{i,t})$*, a Markov pure-strategy sub-game perfect coalition-invariant equilibrium is a collection of the behavioral strategies that satisfy* ***Definition 2*** *and are such that* $g(\psi^*_{1,t}, \psi^*_{2,t}, \psi^*_{3,t})$ *is time-invariant.*

A closely related but very different definition of a pure-strategy coalition-invariant equilibrium is defined on a different game, which only allows for a once-and- for-all a simultaneous coalition decision at the very beginning of period 0. The realized coalition structure cannot be changed afterwards. We can characterize the equilibrium for each of the five possible coalition structures in a recursive manner so the dynamic game can be reduced to an equivalent static game described by the two pay-off matrices in table 1-1 and table 1-2. That is, all the three agents first simultaneously make a coalition choice. Then the realized coalition structure is observed. If full coalition ϖ^f is realized, then all the three agents have no choice but to follow the instruction of the labor supply and consumption for each period from the fictitious planner for that coalition, who solves the Bellman equation (3) in the main text. The consumption is given by (4), and the labor supply is given by (5). The capital evolves according to (6), and the output each period is given by the production function (1). The total sum of the discounted presented value at time zero for each agent is given by $V_i^f(k_0)$ in (8). If no coalition ϖ^n is realized, all the agents will follow (10) and (11), and obtain the value given by (12). If partial coalition $\varpi^{1,2}$ is realized, then all the agents will follow (19), (20), (15), (16) with the values given by (21) and (17). Similar description applies for the other two partial coalition cases. Foreseeing all the entailed values, each of the three agents makes their coalition choices to maximize their own utilities, and the equilibrium is the Nash equilibrium. We allow will focus on the Pareto-dominant Nash equilibrium if there exist multiple equilibria.

B. An Alternative Representation with Contracts.

Consider the following static contracting problem with the following contract space:

$$\Omega(k_0) = \{\varpi^f(k_0), \varpi^{1,2}(k_0), \varpi^{2,3}(k_0), \varpi^{1,3}(k_0), \varpi^n(k_0)\},$$

where ϖ^f refers to the contract of permanent full coalition, $\varpi^{i,j}$ refers to permanent partial coalition between agent i and agent j, $\forall i, j \in \{1,2,3\}, i \neq j$, and ϖ^n refers to no coalition forever. All of these five types of contracts only stipulate who will be permanently in the coalition, for any given initial capital stock, k_0. The contracting only happens at the beginning of the initial period, $t = 0$ and each self-interested agent must choose one and only one contract. Pre-play communication is allowed. If they can't agree on the contract choice, they will be automatically assigned the contract ϖ^n. No random choice over Ω is allowed and we assume full commitment once a contract is finally chosen. It can be shown that the above static contracting problem is fully equivalent to the following sequential contracting problem:

At the initial period, the agents can either choose to remain non-cooperative or choose a permanently binding coalitional contract from $\Omega'(k_0) = \{\varpi^f(k_0), \varpi^{1,2}(k_0), \varpi^{2,3}(k_0), \varpi^{1,3}(k_0)\}$.If no such coalitional contract is chosen, then people will act non-cooperatively in this period and face the same decision problem next period except that the capital stock next period might be different.

Based on the value functions obtained in the main text, we know the endogenous ordering of those contracts for each agent, and hence it becomes a social choice problem with three agents over five contracts. In a democratic society, voting is presumably the most popular way of resolving the issue of interest conflict. However, unless a full coalition is formed, there is always predation between the coalitions.

In the original static contracting problem, Let's denote "is strictly preferred to" by $\succ_i$ for agent i, " is weakly preferred to" by $\geq_i$, and " is indifferent with" by $\sim_i$, for $i = 1,2,3$.

When $\alpha\beta > \zeta_1$, we have the following preference order: $\forall k_0 > 0$,

$$\varpi^f(k_0) \succ_1 \varpi^{2,3}(k_0) \succ_1 \varpi^{1,2}(k_0) \sim_1 \varpi^{1,3}(k_0) \succ_1 \varpi^n(k_0);$$

$$\varpi^f(k_0) \succ_2 \varpi^{1,3}(k_0) \succ_2 \varpi^{2,3}(k_0) \sim_2 \varpi^{1,2}(k_0) \succ_2 \varpi^n(k_0);$$

$$\varpi^f(k_0) \succ_3 \varpi^{1,2}(k_0) \succ_3 \varpi^{2,3}(k_0) \sim_3 \varpi^{1,3}(k_0) \succ_3 \varpi^n(k_0).$$

When $\zeta_2 < \alpha\beta < \zeta_1$, we have $\forall k_0 > 0$,

$$\varpi^{2,3}(k_0) \succ_1 \varpi^f(k_0) \succ_1 \varpi^{1,2}(k_0) \sim_1 \varpi^{1,3}(k_0) \succ_1 \varpi^n(k_0);$$

$$\varpi^{1,3}(k_0) \succ_2 \varpi^f(k_0) \succ_2 \varpi^{2,3}(k_0) \sim_2 \varpi^{1,2}(k_0) \succ_2 \varpi^n(k_0);$$

$$\varpi^{1,2}(k_0) \succ_3 \varpi^f(k_0) \succ_3 \varpi^{2,3}(k_0) \sim_3 \varpi^{1,3}(k_0) \succ_3 \varpi^n(k_0).$$

When $\alpha\beta < \zeta_2$, we have $\forall k_0 > 0$,

$$\varpi^{2,3}(k_0) \succ_1 \varpi^f(k_0) \succ_1 \varpi^n(k_0) \succ_1 \varpi^{1,2}(k_0) \sim_1 \varpi^{1,3}(k_0);$$

$$\varpi^{1,3}(k_0) \succ_2 \varpi^f(k_0) \succ_2 \varpi^n(k_0) \succ_2 \varpi^{2,3}(k_0) \sim_2 \varpi^{1,2}(k_0) \qquad ;$$

$$\varpi^{1,2}(k_0) \succ_3 \varpi^f(k_0) \succ_3 \varpi^n(k_0) \succ_3 \varpi^{2,3}(k_0) \sim_3 \varpi^{1,3}(k_0)$$

Proposition 1'. *Suppose all agents are perfectly identical. When* $\alpha\beta > \zeta_1$, *then* ϖ^f *will be chosen. When* $\zeta_2 < \alpha\beta < \zeta_1$, *any one of the three partial coalition contracts (* $\{\varpi^{1,2}(k_0), \varpi^{2,3}(k_0), \varpi^{1,3}(k_0)\}$ *) will be chosen equally likely. When* $\alpha\beta < \zeta_2$, ϖ^n *will be chosen.*

C. What if Political Power Distribution Follows A Stochastic Process

As is well documented in the political economy literature, the political power distributions among different social groups or different political parties typically fluctuate over time.[23] Moreover, empirical researches show that both income distribution and growth rates demonstrate persistent differences or non-convergence across most of the countries[24]. Our above analysis suggests that the social infrastructure intrinsically determines in the growth rates and consumption inequality. Therefore, it seems natural to explore further the dynamic relationship between the persistence of political power distribution and the persistence of the macroeconomic performance through the change in the coalition structures. For the similar justifications in Dixit, Grossman, and Gul (2000), we assume that the political power distribution (a_1, a_2, a_3) follows an exogenous Markov chain.[25] For parsimony, we assume there are two states with the transitional probability matrix $\begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}$. It's straightforward to extend to n states.

C.1 Full Coalition is Maintained Across Different States

Suppose that the distribution of the political power in both states is sufficiently equal (for example, both states are in the region G in Figure 2). We guess and verify that the full coalition will be the equilibrium in both states. We still focus on the Markov equilibrium.

The Bellman equation for the fictitious planner of the full coalition is now

$$V^f(k, s_j) = \underset{\{c_i, l_i\}_{i=1}^3}{Max} \{ \sum_{i=1}^{3} a_i(j)(\ln c_i - \frac{1}{2} l_i^2) + \beta \sum_{j'=1}^{2} V^f(Ak^\alpha \prod_i l_i^{\mu_i} - \sum_{i=1}^{3} c_i, s_{j'}) \pi_{jj'} \}$$

We will guess and verify that

$$c_{ij} = \lambda_{ij} f(j) \ \forall i = 1,2,3 \ \ j = 1,2 \qquad \text{(C.1.1)}$$

where λ_{ij} is a constant. The first order conditions and the Envelope theorem imply that for each state $j = 1, 2$,

$$\frac{a_{ij}}{\lambda_{ij}} = \theta_j, \forall i, \qquad \text{(C.1.2)}$$

and

$$\lambda_{ij} = \frac{\mu_i}{l_{ij}^2}, \qquad \text{(C.1.3)}$$

and

[23] For more empirical evidences, see the discussions and references in Dixit, Grossman, and Gul (2000).
[24] See Benabou (1996) for a detailed survey on empirical evidence and theoretical discussions.
25 It is surely appealing to ultimately examine the more fundamental causes of the changes in the political power distribution, but as a first step, let's take those changes as exogenous.

$$k' = (1 - \sum_{i=1}^{3} \lambda_{ij}) f(j) . \tag{C.1.4}$$

There are eight unknowns $\{\lambda_{11}, \lambda_{21}, \lambda_{31}, \lambda_{12}, \lambda_{22}, \lambda_{32}, \theta_1, \theta_2\}$. By solving the system of eight equations, we obtain

$$\theta_j = \theta_{j'} = \frac{1}{1-\alpha\beta}, \forall j, j' = 1, 2. \tag{C.1.5}$$

Substituting (C.1.2) and (C.1.5) back into (C.1.1), we get all the six consumption equations. Similarly, we can determine the labor supply in (C.1.3) and the investment in (C.1.4). We can see that the consumption-output ratio is proportional to the political power parameter across all the agents and across both states. Labor supply and hence endogenous growth rate would generally be different across the two states. Interestingly, this means that all the labor supply, consumption and investment ***functions*** will be exactly identical to the economy without uncertainty, which we have derived in Subsection 2.1.

Therefore, the economic growth rates and consumption distributions will also follow a Markov process with the same transitional matrix $\begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}$. This can be interpreted as a stable political institution such as a consolidated democracy or a very strong state of non-democracy,[26] mainly due to the fact that the political power distribution doesn't fluctuate too wildly so that the full coalition structure is time-invariant. The exogenous political power dynamics simply translates into the same dynamics of the macroeconomic performance with the same persistence.

C.2 Coalition Structure Changes Across Different States

Suppose the political power distribution is sufficiently equal in state 1 but extremely unequal in state 2. Inspired by our discussion in the third example in Subsection 3.1.2, we will guess and verify that full coalition is formed in state 1 and no coalition is formed in state 2.

At state s_1, the fictitious planner needs to maximize the full coalition's welfare by solving

$$\begin{aligned} V^f(k, s_1) = \underset{\{c_i, l_i\}_{i=1}^3}{Max} \{ \sum_{i=1}^{3} a_i(1)(\ln c_i - \frac{1}{2} l_i^2) \\ + \beta \left[V^f(Ak^\alpha \prod_i l_i^{\mu_i} - \sum_{i=1}^{3} c_i, s_1)\pi_{11} + \sum_{i=1}^{3} a_i(2) V_i^n (Ak^\alpha \prod_i l_i^{\mu_i} - \sum_{i=1}^{3} c_i, s_2)\pi_{12} \right] \} \end{aligned} \tag{C.2.1}$$

[26] See more discussions on the empirical evidence and relevance about political transitions and the strength of the state, please see Acemoglu (2005), and Acemoglu and Robinson (2006), among others.

For each individual $j \in N$, his own total expected value at the full coalition is equal to

$$V_j^f(k,s_1) = \ln c_j - \frac{1}{2} l_j^2 + \beta\left[V_j^f(Ak^\alpha \prod_i l_i^{\mu_i} - \sum_{i=1}^{3} c_i, s_1)\pi_{11} + V_j^n(Ak^\alpha \prod_i l_i^{\mu_i} - \sum_{i=1}^{3} c_i, s_2)\pi_{12}\right]\}, \tag{C.2.2}$$

where all the consumption and labor supply are solutions to (C.2.1).

At state s_2, for each individual $j \in N$, his own total expected value is equal to

$$V_j^n(k,s_2) = \max_{c_j,l_j}\{\ln c_j(2) - \frac{1}{2} l_j^2(2) + \beta\left[V_j^f(Ak^\alpha \prod_i l_i^{\mu_i}(2) - \sum_{i=1}^{3} c_i(2), s_1)\pi_{21} + V_j^n(Ak^\alpha \prod_i l_i^{\mu_i}(2) - \sum_{i=1}^{3} c_i(2), s_2)\pi_{22}\right]\}, \tag{4.2.3}$$

Again we can guess and verify that each period the consumption is a constant fraction of the current output: $c_j(s) = \lambda_j(s)y(s)$ for each individual $j \in N$ and for each state $s \in \{1,2\}$.

It turns out that at state 2, we have

$$\lambda_1(2) = \lambda_2(2) = \lambda_3(2) = \frac{(1-\alpha\beta\pi_{22})(1-\alpha\beta\pi_{11}) - \alpha^2\beta^2\pi_{12}\pi_{21}}{1-\alpha\beta\pi_{11}+\alpha\beta\pi_{21}} \tag{C.2.4}$$

The labor supply functions are

$$l_j^2(2) = \frac{\mu_j}{\lambda_j(2)}, \forall j \in N \tag{C.2.5}$$

The value functions are

$$V_j^n(k,s_2) = A_j(2)\ln k + B_j(2), \forall j \in N,$$

where $A_j(2) = \dfrac{\alpha}{\lambda_j(2)}$ and $B_j(2)$ is a constant independent of k.

In state 1, we have

$$\lambda_j(1) = a_j(1)\left(\frac{(1-\alpha\beta\pi_{22})(1-\alpha\beta\pi_{11}) - \alpha^2\beta^2\pi_{12}\pi_{21}}{1-\alpha\beta\pi_{11}+\alpha\beta\pi_{21}}\right), \forall j \in N, \tag{C.2.6}$$

and the labor supply functions are

$$l_j^2(1) = \frac{\mu_j}{\lambda_j(1)}, \forall j \in N. \quad \text{(C.2.7)}$$

The value functions are

$$V_j^f(k, s_1) = A_j(1)\ln k + B_j(1), \forall j \in N, \quad \text{(C.2.8)}$$

where $A_j(1) = \frac{\alpha a_j(1)}{\lambda_j(1)}$ and $B_j(1)$ is a constant independent of k.

Hence we can see that $A_j(2) = A_j(2)$, $\forall j \in N$. Similarly, we can compute the value function for each agent at each of the three partial coalitions. Again, we can verify that the coalition decisions are independent of the capital stock and that this Markov change in the coalition structure is indeed the Markov-Perfect Nash equilibrium. This exercise suggests that political transitions between full coalition and no coalition would occur when the political power distribution changes dramatically over time.

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Table 1-1 The Payoff Matrix of the Dynamic Game

Agent 2, when Agent 3 cooperates

Agent 1	Agent 2: Cooperate	Agent 2: not cooperate
Cooperate	$V_1^f(k), V_2^f(k), V_3^f(k)$	$V_1^{1,3}(k), V_2^{1,3}(k), V_3^{1,3}(k)$
not cooperate	$V_1^{2,3}(k), V_2^{2,3}(k), V_3^{2,3}(k)$	$V_1^n(k), V_2^n(k), V_3^n(k)$

when Agent 3 cooperates

Table 1-2 The Payoff Matrix of the Dynamic Game

Agent 1	Agent 2: Cooperate	Agent 2: not cooperate
Cooperate	$V_1^{1,2}(k), V_2^{1,2}(k), V_3^{1,2}(k)$	$V_1^n(k), V_2^n(k), V_3^n(k)$
not cooperate	$V_1^n(k), V_2^n(k), V_3^n(k)$	$V_1^n(k), V_2^n(k), V_3^n(k)$

when Agent 3 does not cooperate

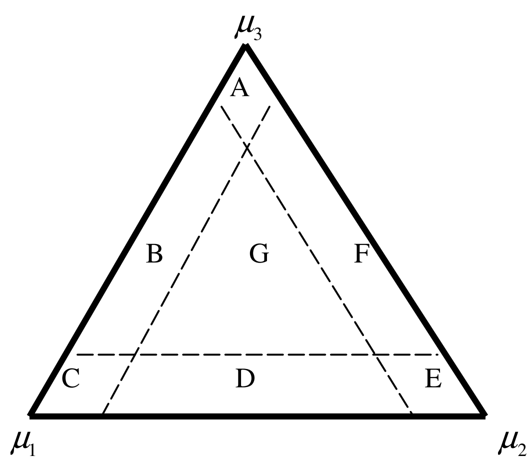

Figure 1: Strategic Behaviors When Political Power is Proportional to Labor Productivity

Illustration: The three rhombi A, C, and E correspond to "No Coalition". The middle equilateral G is "Full Coalition". The three trapezoids B, D, F are "Partial Coalitions".

Table 2: Endogenous Coalition with Heterogeneous Productivities

$\mu_1 = \mu_2$	(0, 0.01837)	(0.01837, 0.08)	(0.08, 0.46)	(0.46, 0.5)
Equilibrium [a]	$\varpi^{1,2}$	ϖ^{n}	ϖ^{f}	$\varpi^{1,2}$
Inequality [b]	1:1:2	1:1:1	1:1:1	1:1:2
Growth Rate [c]	(22)	(13)	(7)	(22)

Note: a. We focus on the Pareto-efficient equilibrium if there exist multiple equilibria. b. It's measured in terms of the consumption ratio. c. It refers to the growth rate on the BGP when $\alpha = 1$. Here (7), (13), and (22) are the indexes for the formula in the main text.

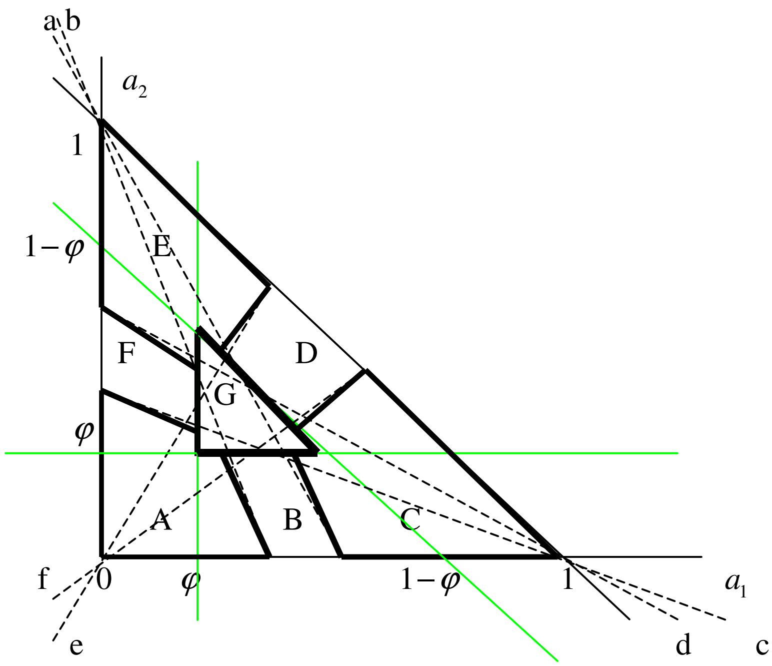

Figure 2: Endogenous coalition with heterogeneous political power