## Online Appendix <br> A1. Derivation of the Demand Function

Suppose the household solves the following problem:

$$
\begin{equation*}
\max _{\left\{Y_{i}\right\}}\left(\sum_{i} D_{i}^{1 / \gamma} Y_{i}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{A1}
\end{equation*}
$$

subject to:

$$
\sum_{i} P_{i} Y_{i} \leq P
$$

We take the budget of the household $P$ to be exogenous. Cost minimization on the part of the representative household implies the demand function:

$$
\begin{equation*}
Y_{i}=D_{i}\left(\frac{P_{i}}{P}\right)^{-\gamma} \tag{A2}
\end{equation*}
$$

The final product in each industry is assembled by competitive firms in each industry that solves:

$$
\begin{equation*}
P_{i} Y_{i}=\min _{\left\{y_{n i}\right\}} \sum_{n \in \Omega_{i}} p_{n i} y_{n i} \tag{A3}
\end{equation*}
$$

subject to:

$$
Y_{i}=\left(\sum_{i} y_{n i}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}
$$

Cost minimization on the part of these competitive firms implies:

$$
\begin{equation*}
y_{n i}=Y_{i}\left(\frac{p_{n i}}{P_{i}}\right)^{-\sigma} \tag{A4}
\end{equation*}
$$

Combining equations (A2) and (A4) implies:

$$
\begin{equation*}
y_{n i}=D_{i}\left(\frac{P_{i}}{P}\right)^{-\gamma}\left(\frac{p_{n i}}{P_{i}}\right)^{-\sigma} \tag{A5}
\end{equation*}
$$

A2. Proof of Proposition 1
Suppose marginal costs of all firms are bounded and non-decreasing. Proposition 1 has the following five parts:

1) If operating independently, firm markups are increasing in a firm's own market share,
2) If operating as a cartel, cartel markups are increasing in total cartel market share with each firm's own market share playing no additional role,
3) Firm markups are higher under cartel decisions than when operating independently,
4) Firm markups are more similar when operating as a cartel than when operating independently,
5) Firm market shares are more similar when operating independently than when operating as a cartel

## PROOF:

Suppose any firm $n$ in industry $i$ weights the profits of the set of firms $S \subset \Omega_{i}$ with constant $\kappa \in[0,1]$. Then their objective is:

$$
\begin{equation*}
\max _{y_{n i}} p\left(y_{n i}\right) y_{n i}-C\left(y_{n i} ; X_{n i}\right)+\kappa \sum_{m \in S}\left[p\left(y_{m i}\right) y_{m i}-C\left(y_{m i} ; X_{m i}\right)\right] \tag{A6}
\end{equation*}
$$

Then for $\mu_{n i}$ defined as price divided by marginal cost and share defined as the firm's revenue divided by the sum of firm revenues in the industry, the firm's first order condition can be rewritten as:

$$
\begin{equation*}
\frac{1}{\mu_{n i}}=1+(1-\kappa) \frac{\partial \log \left(p_{n i}\right)}{\partial \log \left(y_{n i}\right)}+\kappa \sum_{m \in S} \frac{s_{m i}}{s_{n i}} \frac{\partial \log \left(p_{m i}\right)}{\partial \log \left(y_{n i}\right)} \tag{A7}
\end{equation*}
$$

If inverse demand is given by:

$$
\begin{equation*}
p_{n i}=D_{i} y_{n i}^{-1 / \sigma}\left(\sum_{m \in \Omega_{i}} y_{m i}^{1-1 / \sigma}\right)^{\frac{\sigma}{\gamma} \frac{\gamma-1}{\sigma-1}-1} \tag{A8}
\end{equation*}
$$

Then the cross-price elasticities are:

$$
\begin{equation*}
\frac{\partial \log \left(p_{m i}\right)}{\partial \log \left(y_{n i}\right)}=\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right) s_{n i} \tag{A9}
\end{equation*}
$$

The own-price elasticity is:

$$
\begin{equation*}
\frac{\partial \log \left(p_{n i}\right)}{\partial \log \left(y_{n i}\right)}=-\frac{1}{\sigma}+\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right) s_{n i} \tag{A10}
\end{equation*}
$$

Together these imply that:

$$
\begin{equation*}
\frac{1}{\mu_{n i}}=1-\frac{1}{\sigma}+\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right)\left((1-\kappa) s_{n i}+\kappa \sum_{m \in S} s_{m i}\right) \tag{A11}
\end{equation*}
$$

Firms operating independently is the case where $\kappa=0$, so then:

$$
\begin{equation*}
\frac{1}{\mu_{n i}}=1-\frac{1}{\sigma}+\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right) s_{n i} \tag{A12}
\end{equation*}
$$

This implies result 1 , when $\sigma>\gamma$. Likewise, if firms are operating as a perfect cartel, then $\kappa=1$ :

$$
\begin{equation*}
\frac{1}{\mu_{n i}}=1-\frac{1}{\sigma}+\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right) \sum_{m \in S} s_{m i} \tag{A13}
\end{equation*}
$$

This immediately implies the second result. Moreover, equations (A12) and (A13) together imply the fourth result, as cartels have no variation in markups (even if they have variation in market shares) while independent firms have markups that vary with their shares.

To compare firms in a cartel to those operating independently, we construct an artificial single firm that is equivalent to the cartel. That is, suppose $\kappa=1$ so that the cartel solves:

$$
\begin{equation*}
\max _{\left\{y_{m i}\right\}} \sum_{m \in S}\left(p_{m i} y_{m i}-C\left(y_{m i} ; X_{m i}\right)\right) \tag{A14}
\end{equation*}
$$

where $p_{m i}$ is given by (A8). Now define a cartel aggregate of production:

$$
\begin{equation*}
Y=\left(\sum_{m \in S} y_{m i}^{1-1 / \sigma}\right)^{\frac{\sigma}{\sigma-1}} \tag{A15}
\end{equation*}
$$

Let $\tilde{C}(Y)$ be the cost function of the cartel defined as:

$$
\begin{gather*}
\tilde{C}(Y)=\min _{\left\{y_{m i}\right\}} \sum_{m \in S} C\left(y_{m i} ; X_{m i}\right)  \tag{A16}\\
\text { subject to: } \quad Y=\left(\sum_{m \in S} y_{m i}^{1-1 / \sigma}\right)^{\frac{\sigma}{\sigma-1}}
\end{gather*}
$$

Then the following problem is equivalent to (A14):

$$
\begin{equation*}
\max _{Y} D_{i} Y^{1-1 / \sigma}\left(Y^{1-1 / \sigma}+\sum_{\substack{n \notin S \\ 3}} y_{n i}^{1-1 / \sigma}\right)^{\frac{\sigma}{\gamma} \frac{\gamma-1}{\sigma-1}-1}-\tilde{C}(Y) \tag{A17}
\end{equation*}
$$

First notice that the Envelope Theorem applied to the problem in (A16):

$$
\begin{equation*}
\forall m \in S, \quad \tilde{C}^{\prime}(Y)=\lambda=\frac{C^{\prime}\left(y_{m i} ; X_{m i}\right)}{y_{m i}^{-1 /{ }^{\prime}} Y^{1 / \sigma}} \tag{A18}
\end{equation*}
$$

Then we can relate the size of the cartel to the cost of the cartel's production.
LEMMA 1: Consider a cartel made up of in $T \subset S$. Then for every level of production $Y$, the marginal cost in the cartel composed of $T$ is strictly higher than in the cartel composed of $S$.
To prove this lemma, suppose $y_{m i}^{T}$ is how much firm $m$ produces when part of the cartel composed of $T$ and $y_{m i}^{S}$ is how much the same firm produces when part of the cartel composed of $S$. Then for any given $Y$ it must be the case that:

$$
y_{m i}^{S}<y_{m i}^{T} \Longrightarrow \frac{C^{\prime}\left(y_{m i}^{S} ; X_{m i}\right)}{y_{m i}^{S-1 / \sigma} Y^{1 / \sigma}}<\frac{C^{\prime}\left(y_{m i}^{T} ; X_{m i}\right)}{y_{m i}^{T-1 / \sigma} Y^{1 / \sigma}} \Longrightarrow \tilde{C}^{\prime}(Y)<\tilde{C}^{T^{\prime}}(Y)
$$

where the second implication follows from the fact that all firms have nondecreasing marginal costs. The first inequality follows from bounded marginal costs and Inada conditions in the aggregation of individual firm production to cartel-level production. Therefore, if more firms are added to a cartel, marginal costs for the cartel are reduced for every level of output.
Given this lemma, notice that as a cartel grows, the markup that the cartel charges strictly increases. This follows immediately from that fact that, given the lemma, marginal costs decline so cartel production increases, and as another firm from within the same industry is brought into the cartel, that firm's production is no longer counted in the denominator when computing the cartel's market share. Therefore, the cartel's market share strictly increases as more firms are added. Hence, by (A13), the markup charged by the cartel increases.

A special case of this result is part 3 of Proposition 1. If a firm is operating outside of an existing cartel then is brought into it, the new cartel would have strictly higher markups than either the original cartel or the formerly independent firm.
To demonstrate the last result, consider any two firms $n$ and $m$ within the same cartel. Manipulating (A18) gives:

$$
\begin{equation*}
\frac{C^{\prime}\left(y_{m i} ; X_{m i}\right)}{C^{\prime}\left(y_{n i} ; X_{n i}\right)}=\left(\frac{y_{m i}}{y_{n i}}\right)^{-\frac{1}{\sigma}}=\left(\frac{s_{m i}}{s_{n i}}\right)^{\frac{1}{1-\sigma}} \tag{A19}
\end{equation*}
$$

Then consider two other firms $v$ and $w$ that are operating independently. Then the relationship between marginal cost and market share is:

$$
\begin{equation*}
\frac{C^{\prime}\left(y_{v i} ; X_{v i}\right)}{C^{\prime}\left(y_{w i} ; X_{w i}\right)}=\left(\frac{s_{v i}}{s_{w i}}\right)^{\frac{1}{1-\sigma}} \frac{1-1 / \sigma+(1 / \sigma-1 / \gamma) s_{v i}}{1-1 / \sigma+(1 / \sigma-1 / \gamma) s_{w i}} \tag{A20}
\end{equation*}
$$

Suppose these two pairs of firms have the same relative marginal costs. Then:

$$
\begin{gather*}
\frac{C^{\prime}\left(y_{m i} ; X_{m i}\right)}{C^{\prime}\left(y_{n i} ; X_{n i}\right)}=\frac{C^{\prime}\left(y_{v i} ; X_{v i}\right)}{C^{\prime}\left(y_{w i} ; X_{w i}\right)} \Longrightarrow  \tag{A21}\\
\left(\frac{s_{m i}}{s_{n i}}\right)^{\frac{1}{1-\sigma}}=\left(\frac{s_{v i}}{s_{w i}}\right)^{\frac{1}{1-\sigma}} \frac{1-1 / \sigma+(1 / \sigma-1 / \gamma) s_{v i}}{1-1 / \sigma+(1 / \sigma-1 / \gamma) s_{w i}}
\end{gather*}
$$

Without loss, if firms $v$ and $m$ have relatively high costs, then:

$$
\begin{gather*}
\frac{C^{\prime}\left(y_{m i} ; X_{m i}\right)}{C^{\prime}\left(y_{n i} ; X_{n i}\right)}=\frac{C^{\prime}\left(y_{v i} ; X_{v i}\right)}{C^{\prime}\left(y_{w i} ; X_{w i}\right)}>1 \Longrightarrow  \tag{A22}\\
\frac{1-1 / \sigma+(1 / \sigma-1 / \gamma) s_{v i}}{1-1 / \sigma+(1 / \sigma-1 / \gamma) s_{w i}}>1 \Longrightarrow \frac{s_{n i}}{s_{m i}}>\frac{s_{w i}}{s_{v i}}
\end{gather*}
$$

Therefore, independently operating firms have wider variation in market shares conditional on marginal cost than do firms operating as a cartel. This completes the proof.

## A3. Simulation of Model with Shocks to Demand and Costs

We now consider a version of the model where some uncertainty in costs or demand is realized after production choices are made. Firm $i$ in industry $j$ located in region $k$ in year $t$ solves the following problem:

$$
\max _{l_{i j k t}} \int_{S_{\varepsilon}} \int_{S_{\rho}}\left[(1-\kappa) \pi_{i j k t}(l, \varepsilon, \rho)+\kappa \sum_{m \in \omega_{j k t}} \pi_{m j k t}(l, \varepsilon, \rho)\right] d F(\varepsilon) d G(\rho)
$$

where:

$$
\pi_{i j k t}(l, \varepsilon, \rho)=D_{j}\left(\varepsilon_{i j k t} l_{i j k t}^{1 / \eta}\right)^{1-1 / \sigma}\left(\sum_{m \in \Omega_{j t}}\left(\varepsilon_{m j k t} l_{m j k t}^{1 / \eta}\right)^{1-1 / \sigma}\right)^{\frac{\sigma}{\gamma} \frac{\gamma-1}{\sigma-1}-1}-\rho_{i j k t} \frac{l_{i j k t}}{z_{i j k t}}
$$

Here $\varepsilon$ is the vector of demand shocks, $\rho$ is the vector of cost shocks, and $l$ is the vector of production choices. The set of firms operating in industry $j$ at time $t$ is $\Omega_{j t}$, and its subset of firms operating within region $k$ is $\omega_{j k t}$. For any given firm, $z_{i j k t}$ is the component of their costs that is known before production decisions are made. Without heterogeneity in this, there would be no heterogeneity in $l_{i j k t}$. The parameter $\eta$ allows for curvature in the cost function.
Notice that $F$ and $G$ are probability distributions over vectors, and we will consider covariance at the cluster, industry and year levels.

The first order condition implies:

$$
\begin{gathered}
\int_{S_{\rho}} \frac{\eta \rho_{i j k t} l_{i j k t}^{1-1 / \eta}}{z_{i j k t}} d G(\rho)= \\
=\int_{S_{\varepsilon}} p_{i j k t}(l, \varepsilon)\left[\frac{\sigma-1}{\sigma}+\left(\frac{1}{\sigma}-\frac{1}{\gamma}\right) \frac{\kappa\left(\varepsilon_{i j k t} l_{i j k t}^{1 / \eta}\right)^{1-1 / \sigma}+(1-\kappa) \sum_{n \in \omega_{j k t}}\left(\varepsilon_{n j k t} l_{n j k t}^{1 / \eta}\right)^{1-1 / \sigma}}{\sum_{m \in \Omega_{j t}}\left(\varepsilon_{m j k t}^{1 / \eta} l_{m j k t}^{1-1 / \sigma}\right)^{1 / \sigma}}\right] d F(\varepsilon)
\end{gathered}
$$

where:

$$
p_{i j k t}(l, \varepsilon)=D_{j} \varepsilon_{i j k t}^{1-1 / \sigma} l_{i j k t}^{-1 / \eta \sigma}\left(\sum_{m \in \Omega_{j t}}\left(\varepsilon_{m j k t} l_{m j k t}\right)^{1 / \eta(1-1 / \sigma)}\right)^{\frac{\sigma}{\gamma} \frac{\gamma-1}{\sigma-1}-1}
$$

Firms face a variety of shocks at different levels:

$$
\begin{gathered}
\varepsilon_{i j k t}=\nu_{1} \varepsilon_{t}^{1}+\nu_{2} \varepsilon_{j t}^{2}+\nu_{3} \varepsilon_{i j k t}^{3}+\nu_{4} \varepsilon_{j k t}^{4}+\nu_{5} \varepsilon_{k t}^{5} \\
\rho_{i j k t}=\mu_{1} \rho_{t}^{1}+\mu_{2} \rho_{j t}^{2}+\mu_{3} \rho_{i j k t}^{3}+\mu_{4} \rho_{j k t}^{4}+\mu_{5} \rho_{k t}^{5}
\end{gathered}
$$

Therefore, we can separately analyze shocks at different levels.

## Computational Implementation

The simulated dataset has $T$ years, $J$ industries and $K$ regions. Every industry-region-year has $I$ firms within it. The vectors $\varepsilon$ and $\rho$ are therefore of length $I \times J \times K \times T$. First, both $\varepsilon$ and $\rho$ are simulated $M$ times. Then a vector $L$ is drawn. Then $L$ is input as the vector of production choices of firms. Using the first order condition, we then solve for the vector $Z$ of anticipated costs that rationalizes the vector $L$. Together, $Z, L$, and the realization of shocks implies markups (using the method of De Loecker and Warzynski) and market shares for each firm. Then, for each realization, the regression described in the paper is run on the simulated data. This is done $M$ times.
For these results we choose $\sigma=5, \gamma=3$, and $\kappa=0.3$. We set $T=11, J=5$, $K=8, I=10$ and $M=1000$. We assume that the $\log$ of each shock is a standard normal random variable.

Effects of Shocks: Comparative Statics

First we look at the effects of all twelve types of shocks individually. The table below presents the results of setting $\mu_{1}=\ldots=\mu_{5}=\nu_{1}=\ldots=\nu_{5}=0$, then individually setting each to 1 .

In each iteration of the simulation we run the following regression:

$$
\frac{1}{\operatorname{markup}_{i j k t}}=\alpha+\beta_{1} s_{i j k t}+\beta_{2} c_{j k t}+\delta_{i j k t}
$$

where:

$$
\begin{gathered}
s_{i j k t}=\frac{\left(\varepsilon_{i j k t} y_{i j k t}\right)^{1-1 / \sigma}}{\sum_{m \in \Omega_{j t}}\left(\varepsilon_{m j k t} y_{m j k t}\right)^{1-1 / \sigma}} \\
c_{j k t}=\sum_{l \in \omega_{j k t}} s_{l j k t}
\end{gathered}
$$

Here we present the simulated moments of $\hat{\kappa}$ defined by:

Table A1-Simulation Results: Ex Post Shocks

|  | No Fixed Effects |  |  |  | Region-Year and Firm FEs |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cost Shocks: | Avg. $\hat{\kappa}$ | St. Dev. $\hat{\kappa}$ | Adj. $R^{2}$ | Avg. $\hat{\kappa}$ | St. Dev. $\hat{\kappa}$ | Adj. $R^{2}$ |  |
| Year | 0.3000 | 0.0075 | -0.0006 | 0.2995 | 0.0171 | 0.9999 |  |
| Industry-Year | 0.3000 | 0.0015 | 0.0030 | 0.3000 | 0.0024 | 0.1719 |  |
| Firm-Year | 0.0120 | 0.0562 | 0.0093 | 0.0044 | 0.0577 | 0.0130 |  |
| Cluster-Year | 0.9759 | 0.0059 | 0.0805 | 0.9982 | 0.0059 | 0.2628 |  |
| Region-Year | 0.2227 | 0.0332 | 0.0175 | 0.3178 | 0.0253 | 0.6229 |  |
| Demand Shocks: | Avg. $\hat{\kappa}$ | St. Dev. $\hat{\kappa}$ | Adj. $R^{2}$ | Avg. $\hat{\kappa}$ | St. Dev. $\hat{\kappa}$ | Adj. $R^{2}$ |  |
| Year | 0.3000 | 0.0065 | -0.0006 | 0.3000 | 0.0145 | 0.9999 |  |
| Industry-Year | 0.2999 | 0.0086 | -0.0006 | 0.2998 | 0.0149 | 0.1650 |  |
| Firm-Year | 0.0652 | 6.7790 | -0.0001 | 0.0199 | 5.3935 | -0.0003 |  |
| Cluster-Year | 1.1084 | 24.2151 | 0.0016 | 0.8891 | 5.2687 | 0.1846 |  |
| Region-Year | 0.0179 | 23.5850 | 0.0021 | 0.3018 | 0.0215 | 1.0000 |  |
|  | Firm FEs |  |  |  |  |  |  |
| Cost Shocks: | Avg. $\hat{\kappa}$ | St. Dev. $\hat{\kappa}$ | Adj. $R^{2}$ | Avg. $\hat{\kappa}$ | St. Dev. $\hat{\kappa}$ | Adj. $R^{2}$ |  |
| Year | 0.3003 | 0.0124 | -0.1004 | 0.3004 | 0.0098 | 1.000 |  |
| Industry-Year | 0.3000 | 0.0020 | -0.0147 | 0.3000 | 0.0016 | 0.1694 |  |
| Firm-Year | 0.0035 | 0.0499 | 0.0125 | 0.0074 | 0.0605 | 0.0094 |  |
| Cluster-Year | 0.9986 | 0.0053 | 0.0984 | 0.9752 | 0.0066 | 0.2483 |  |
| Region-Year | 0.2253 | 0.0306 | -0.0113 | 0.3101 | 0.0222 | 0.5584 |  |
| Demand Shocks: | Avg. $\hat{\kappa}$ | St. Dev. $\hat{\kappa}$ | Adj. $R^{2}$ | Avg. $\hat{\kappa}$ | St. Dev. $\hat{\kappa}$ | Adj. $R^{2}$ |  |
| Year | 0.2995 | 0.0124 | -0.1004 | 0.2999 | 0.0085 | 1.0000 |  |
| Industry-Year | 0.2998 | 0.0126 | -0.0187 | 0.2998 | 0.0111 | 0.1711 |  |
| Firm-Year | -0.2294 | 15.1836 | 0.0002 | -0.1389 | 6.0075 | -0.0002 |  |
| Cluster-Year | 0.7296 | 8.6585 | -0.0002 | 1.1787 | 2.6894 | 0.1893 |  |
| Region-Year | -1.3071 | 53.0598 | 0.0092 | 0.3004 | 0.0110 | 0.9999 |  |

$$
\hat{\kappa} \equiv \frac{\beta_{2}}{\beta_{1}+\beta_{2}}
$$

The results from these experiments are given in Table A.A3. We provide four sets of results based on the set of fixed effects considered, and for each case we provide the average and standard deviation of $\kappa$ across the 1000 simulations. We also provide the adjusted $R^{2}$ averaged across the 1000 simulations.

These results demonstrate two important things to help understand how our estimates of $\kappa$ could be biased. Firm-year shocks bias estimates of $\kappa$ downward, and cluster-year and region-year shocks bias estimates upward. The region-year shocks can be mitigated with region-year fixed effects: the bias is almost eliminated for cost shocks and is less severe for demand shocks. In the other cases, the adjusted $R^{2}$ of the model can fall considerably, but we see little evidence of bias in estimates of $\kappa$.

## Calibrated Example

The previous subsection demonstrates that the most serious bias arises when ex post shocks are at the firm-year and cluster-year level. We now repeat the numerical exercise from the previous section but now we parameterize the model to replicate the results of our baseline results in column 7 of Table 4. As in that regression, we include firm and year fixed effects and cluster standard errors at the firm level. We consider ex post shocks to productivity at the firm-year level and the cluster-year level, and we include measurement error at the firm-level. We also have idiosyncratic firm-year ex ante shocks. Each shock is assumed to be log-normal.
We calibrate six parameters: the variance of the three shocks, $\gamma, \sigma$, and $\kappa$. We match six moments: the coefficient estimate on the firm's own share and on the cluster's share, the standard errors on the firm's own share and on the cluster share, the average markup, and the regression's within- $R^{2}$.
The calibrated value of $\kappa$ is 0.29 , while the value in the model, as in the data, is 0.28 . This demonstrates that, in this case, we actually underestimate the degree of collusion with our procedure relative to its true value. The calibrated standard deviation of the firm-year productivity shock is 0.011 while that of the cluster-year shock is 0.009 . Our estimate of $\sigma$ is 4.57 and $\gamma$ is 2.74 . The standard deviation of the measurement error is 0.044 .

## Departure from CES Demand

Next we consider the case where the demand system is instead given by:

$$
\begin{equation*}
q_{i j k t}=\left(\frac{p_{i j k t}+\bar{p}}{P_{j t}}\right)^{-\sigma}\left(\frac{P_{j t}}{P_{t}}\right)^{-\gamma} \tag{A23}
\end{equation*}
$$

Proceeding with the same simulation technique as above, we consider the case where there are no ex post shocks and vary the magnitude of $\bar{p}$.


The results are summarized below in Figures A1 and A2. As the value of $\bar{p}$ varies, as shown on the horizontal axis in both figures, on average our measure of $\kappa$ will be affected monotonically as shown in Figure A1. As before, the true value of $\kappa$ in this simulation is equal to 0.3 . Figure A2 shows that this bias is entirely due to bias in the coefficient on firms' own shares. In fact, the coefficient on cluster shares is unbiased by $\bar{p}$.
This supports our conclusion that a non-CES demand system of this type affects our estimate of the magnitude of collusion. However, if we interpret the t-test of whether or not the coefficient on the cluster share is positive to be a test of collusion, that test is unaffected by non-CES demand systems of this form.

## Measurement Error

Finally, we consider the case where revenues are measured with error. We proceed as before, but now instead of unanticipated shocks, we study the effect of increases in the variances of the measurement error.
Following the parameterization in the first simulation exercise, A.A3 shows the effects of measurement error. In the "Idiosyncratic" columns, we assume that measurement error has no correlation across firms. In the "Cluster" columns, we consider the extreme case of correlation within clusters where measurement errors are equal in all firms of the same cluster.

Figure A2. Varying Non-Homotheticity: Coefficient Estimates

Table A2-Effects of Measurement Error

|  | Measurement Error, Idiosyncratic |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Var. of Error | Avg. $\hat{\kappa}$ | St. Dev. $\hat{\kappa}$ | Avg. $\hat{\beta}_{1}$ | Avg. $\hat{\beta}_{2}$ |  |  |
| 0.1 | 0.2763 | 0.0520 | -0.0899 | -0.0337 |  |  |
| 0.2 | 0.2135 | 0.0781 | -0.1280 | -0.0333 |  |  |
| 0.3 | 0.1563 | 0.0812 | -0.1926 | -0.0335 |  |  |
| 0.4 | 0.1170 | 0.0746 | -0.2736 | -0.0341 |  |  |
| 0.5 | 0.0840 | 0.0672 | -0.3846 |  |  | -0.0328 |
|  | Measurement Error, Cluster |  |  |  |  |  |
| Var. of Error | Avg. $\hat{\kappa}$ | St. Dev. $\hat{\kappa}$ | Avg. $\hat{\beta}_{1}$ | Avg. $\hat{\beta}_{2}$ |  |  |
| 0.1 | 0.3602 | 0.0855 | -0.0777 | -0.0457 |  |  |
| 0.2 | 0.4954 | 0.1102 | -0.0775 | -0.0826 |  |  |
| 0.3 | 0.6322 | 0.1116 | -0.0771 | -0.1469 |  |  |
| 0.4 | 0.7253 | 0.0869 | -0.0769 | -0.2236 |  |  |
| 0.5 | 0.8033 | 0.0599 | -0.0761 | -0.3391 |  |  |

Table A.1: Appendix Table-Placebo Test Using Affiliate Sample

|  | Dependent Variable: $\frac{1}{\mu_{\text {nit }}}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) <br> Province | $\begin{gathered} \hline(3) \\ \text { City } \end{gathered}$ | $\begin{gathered} (4) \\ \text { County } \end{gathered}$ | (5) <br> Province | $\begin{gathered} \hline(6) \\ \text { City } \end{gathered}$ | (7) County | (8) <br> Province | $\begin{aligned} & \hline(9) \\ & \text { City } \end{aligned}$ | $\begin{gathered} (10) \\ \text { County } \end{gathered}$ |
| Firm's share | $\begin{gathered} \hline-0.009 \\ (0.060) \end{gathered}$ |  |  |  | $\begin{gathered} -0.029 \\ (0.063) \end{gathered}$ | $\begin{gathered} -0.016 \\ (0.067) \end{gathered}$ | $\begin{gathered} -0.038 \\ (0.080) \end{gathered}$ | $\begin{gathered} -0.066 \\ (0.065) \end{gathered}$ | $\begin{gathered} \hline-0.048 \\ (0.070) \end{gathered}$ | $\begin{gathered} \hline-0.064 \\ (0.082) \end{gathered}$ |
| Region's share |  | $\begin{gathered} 0.019 \\ (0.021) \end{gathered}$ | $\begin{gathered} 0.004 \\ (0.030) \end{gathered}$ | $\begin{gathered} 0.013 \\ (0.040) \end{gathered}$ | $\begin{gathered} 0.021 \\ (0.022) \end{gathered}$ | $\begin{gathered} 0.007 \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.030 \\ (0.054) \end{gathered}$ | $\begin{gathered} 0.028 \\ (0.023) \end{gathered}$ | $\begin{gathered} 0.009 \\ (0.035) \end{gathered}$ | $\begin{gathered} 0.026 \\ (0.056) \end{gathered}$ |
| SEZ*Firm's share |  |  |  |  |  |  |  | $\begin{gathered} 0.094 \\ (0.137) \end{gathered}$ | $\begin{gathered} 0.100 \\ (0.137) \end{gathered}$ | $\begin{gathered} 0.100 \\ (0.137) \end{gathered}$ |
| SEZ*Region's share |  |  |  |  |  |  |  | $\begin{gathered} 0.019 \\ (0.039) \end{gathered}$ | $\begin{gathered} 0.022 \\ (0.039) \end{gathered}$ | $\begin{gathered} 0.023 \\ (0.039) \end{gathered}$ |
| SEZ Dummy |  |  |  |  |  |  |  | $\begin{gathered} 0.004 \\ (0.005) \end{gathered}$ | $\begin{gathered} 0.004 \\ (0.005) \end{gathered}$ | $\begin{gathered} 0.004 \\ (0.005) \end{gathered}$ |
| Year FEs | YES | YES | YES | YES | YES | YES | YES | YES | YES | YES |
| Firm FEs | YES | YES | YES | YES | YES | YES | YES | YES | YES | YES |
| Observations | 24780 | 24780 | 24780 | 24780 | 24780 | 24780 | 24780 | 20686 | 20686 | 20686 |
| Overall $R^{2}$ | 0.010 | 0.012 | 0.010 | 0.010 | 0.012 | 0.010 | 0.010 | 0.010 | 0.009 | 0.009 |

Notes: Robust standard errors clustered at firm level in parentheses. Significance: ${ }^{* * *}: 1 \%,{ }^{* *}: 5 \%,{ }^{*}: 10 \%$. Regions are defined at various aggregation levels, including province (in specifications 2, 5, and 8), city (in specifications 3, 6, and 9), and county (in specifications 4 , 7 , and 10). All specifications are regressions weighted by the number of observations for each two-digit CIC sector production function estimation reported (following De Loecker et al. 2014). All regressions include a constant term.
Table A.2: Appendix Table-Placebo Test Using SOE Sample

|  | Dependent Variable: $\frac{1}{\mu_{\text {nit }}}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) <br> Province | $\begin{gathered} (3) \\ \text { City } \end{gathered}$ | $\begin{gathered} (4) \\ \text { County } \end{gathered}$ | $\begin{gathered} (5) \\ \text { Province } \end{gathered}$ | $\begin{gathered} \hline(6) \\ \text { City } \end{gathered}$ | $\begin{gathered} (7) \\ \text { County } \end{gathered}$ | (8) <br> Province | $\begin{gathered} (9) \\ \text { City } \end{gathered}$ | $\begin{gathered} \hline(10) \\ \text { County } \end{gathered}$ |
| Firm's share | $\begin{aligned} & \hline-0.047 \\ & (0.055) \end{aligned}$ |  |  |  | $\begin{gathered} -0.029 \\ (0.056) \end{gathered}$ | $\begin{aligned} & -0.066 \\ & (0.058) \end{aligned}$ | $\begin{gathered} -0.061 \\ (0.070) \end{gathered}$ | $\begin{gathered} -0.078 \\ (0.054) \end{gathered}$ | $\begin{aligned} & -0.101^{*} \\ & (0.060) \end{aligned}$ | $\begin{aligned} & -0.147^{*} \\ & (0.076) \end{aligned}$ |
| Region's share |  | $\begin{gathered} -0.024^{*} \\ (0.013) \end{gathered}$ | $\begin{gathered} 0.005 \\ (0.023) \end{gathered}$ | $\begin{gathered} -0.020 \\ (0.037) \end{gathered}$ | $\begin{gathered} -0.021 \\ (0.014) \end{gathered}$ | $\begin{gathered} 0.020 \\ (0.025) \end{gathered}$ | $\begin{gathered} 0.013 \\ (0.048) \end{gathered}$ | $\begin{gathered} -0.010 \\ (0.016) \end{gathered}$ | $\begin{gathered} 0.016 \\ (0.028) \end{gathered}$ | $\begin{gathered} 0.060 \\ (0.055) \end{gathered}$ |
| SEZ*Firm's share |  |  |  |  |  |  |  | $\begin{gathered} 0.052 \\ (0.120) \end{gathered}$ | $\begin{gathered} 0.050 \\ (0.120) \end{gathered}$ | $\begin{gathered} 0.050 \\ (0.119) \end{gathered}$ |
| SEZ*Region's share |  |  |  |  |  |  |  | $\begin{gathered} -0.014 \\ (0.039) \end{gathered}$ | $\begin{gathered} -0.016 \\ (0.039) \end{gathered}$ | $\begin{gathered} -0.016 \\ (0.039) \end{gathered}$ |
| SEZ dummy |  |  |  |  |  |  |  | $\begin{gathered} -0.000 \\ (0.004) \end{gathered}$ | $\begin{gathered} -0.000 \\ (0.004) \end{gathered}$ | $\begin{gathered} -0.000 \\ (0.004) \end{gathered}$ |
| Year FEs | YES | YES | YES | YES | YES | YES | YES | YES | YES | YES |
| Firm FEs | YES | YES | YES | YES | YES | YES | YES | YES | YES | YES |
| Observations | 106434 | 106434 | 106434 | 106434 | 106434 | 106434 | 106434 | 69608 | 69608 | 69608 |
| Overall $R^{2}$ | 0.034 | 0.030 | 0.032 | 0.033 | 0.031 | 0.033 | 0.034 | 0.034 | 0.035 | 0.035 |
| Notes: Robust standard aggregation levels, inclu 7, and 10). All specific estimation reported (fol |  | ered at firm ce (in specifi gressions we oecker et al. |  |  |  |  |  | Regions a nd county IC sector | defined specific oduction | various ions 4 , unction |

Table A.3: Appendix Table-Rauch Product Classification Results

|  | Dependent Variable: $\frac{1}{\mu_{\text {nit }}}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) homo/ref | (2) diff. | (3) overall | (4) homo/ref | (5) diff. | (6) overall |
| Firm's Share | $\begin{gathered} \hline-0.167^{* *} \\ (0.080) \end{gathered}$ | $\begin{gathered} \hline-0.074^{* * *} \\ (0.023) \end{gathered}$ | $\begin{gathered} -0.127^{* *} \\ (0.051) \end{gathered}$ | $\begin{aligned} & \hline-0.141 \\ & (0.202) \end{aligned}$ | $\begin{aligned} & \hline-0.046^{*} \\ & (0.024) \end{aligned}$ | $\begin{gathered} \hline-0.179^{* * *} \\ (0.040) \end{gathered}$ |
| Region's Share | $\begin{gathered} -0.076^{* * *} \\ (0.011) \end{gathered}$ | $\begin{gathered} -0.026^{* * *} \\ (0.008) \end{gathered}$ | $\begin{gathered} -0.067^{* * *} \\ (0.010) \end{gathered}$ | $\begin{gathered} -0.296^{* * *} \\ (0.089) \end{gathered}$ | $\begin{gathered} -0.016^{*} \\ (0.009) \end{gathered}$ | $\begin{gathered} -0.070^{* * *} \\ (0.009) \end{gathered}$ |
| Differentiated X Firm's Share |  |  | $\begin{gathered} 0.042 \\ (0.056) \end{gathered}$ |  |  | $\begin{gathered} 0.126^{* * *} \\ (0.044) \end{gathered}$ |
| Differentiated X Region's Share |  |  | $\begin{gathered} 0.045^{* * *} \\ (0.013) \end{gathered}$ |  |  | $\begin{gathered} 0.056^{* * *} \\ (0.012) \end{gathered}$ |
| Differentiated Dummy |  |  | $\begin{gathered} -0.002 \\ (0.001) \end{gathered}$ |  |  | $\begin{aligned} & -0.001 \\ & (0.001) \end{aligned}$ |
| Year FEs | YES | YES | YES | YES | YES | YES |
| Firm FEs | YES | YES | YES | YES | YES | YES |
| Observations | 273327 | 935702 | 1279149 | 75692 | 642132 | 1279149 |
| Overall $R^{2}$ | 0.036 | 0.024 | 0.027 | 0.015 | 0.019 | 0.027 |

Notes: Robust standard errors clustered at firm level in parentheses. Significance: ${ }^{* * *}: 1 \%,{ }^{* *}: 5 \%,{ }^{*}: 10 \%$. Specifications 1-3 refer to product classification using "most frequent" principle; specifications 4-6 refer to product classification using "pure" principle. All specifications are regressions weighted by the number of observations for each two-digit CIC sector production function estimation reported (following De Loecker et al. 2014). All regressions include a constant term.
Table A.4: Appendix Table-Instrumental Variable Estimation Results Using Low CV Deciles

|  | Dependent Variable: $\frac{1}{\mu_{\text {nit }}}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) <br> Province | $\begin{gathered} (2) \\ \text { City } \end{gathered}$ | (3) County | (4) <br> Province | $\begin{gathered} (5) \\ \text { City } \end{gathered}$ | (6) County | (7) <br> Province | $\begin{gathered} (8) \\ \text { City } \end{gathered}$ | (9) <br> County |
| Firm's share | $\begin{gathered} 0.007 \\ (0.130) \end{gathered}$ | $\begin{gathered} -0.383^{* * *} \\ (0.114) \end{gathered}$ | $\begin{gathered} -0.224^{* * *} \\ (0.052) \end{gathered}$ |  |  |  | $\begin{gathered} 0.100 \\ (0.178) \end{gathered}$ | $\begin{aligned} & -0.229^{*} \\ & (0.121) \end{aligned}$ | $\begin{gathered} -0.113^{* * *} \\ (0.037) \end{gathered}$ |
| Region's share |  |  |  | $\begin{gathered} 0.010 \\ (0.016) \end{gathered}$ | $\begin{gathered} -0.042^{* * *} \\ (0.011) \end{gathered}$ | $\begin{gathered} -0.057^{* * *} \\ (0.016) \end{gathered}$ | $\begin{gathered} 0.018 \\ (0.022) \end{gathered}$ | $\begin{gathered} -0.030^{* * *} \\ (0.012) \end{gathered}$ | $\begin{gathered} -0.048^{* * *} \\ (0.017) \end{gathered}$ |
| Year FEs | YES | YES | YES | YES | YES | YES | YES | YES | YES |
| Firm FEs | YES | YES | YES | YES | YES | YES | YES | YES | YES |
| Observations | 258745 | 154012 | 187050 | 258745 | 154012 | 187050 | 258745 | 154012 | 187050 |
| Overall $R^{2}$ | 0.031 | 0.014 | 0.023 | 0.031 | 0.015 | 0.023 | 0.031 | 0.015 | 0.023 |

[^0]Table A.5: Appendix Table-Robustness

|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: Dependent Variable $=1 / \mu_{\text {nit }}$ |  |  |  |  |  |  |  |  |
| Firm's Share | $\begin{gathered} \hline-0.143^{* * *} \\ (0.021) \end{gathered}$ |  | $\begin{gathered} \hline-0.099^{* * *} \\ (0.022) \end{gathered}$ | $\begin{gathered} \hline-0.163^{* * *} \\ (0.027) \end{gathered}$ | $\begin{gathered} -0.083^{* * *} \\ (0.017) \end{gathered}$ |  | $\begin{gathered} -0.046^{* *} \\ (0.018) \end{gathered}$ | $\begin{gathered} \hline-0.089^{* * *} \\ (0.023) \end{gathered}$ |
| Region's Share |  | $\begin{gathered} -0.053^{* * *} \\ (0.006) \end{gathered}$ | $\begin{gathered} -0.040^{* * *} \\ (0.006) \end{gathered}$ | $\begin{gathered} -0.031^{* * *} \\ (0.007) \end{gathered}$ |  | $\begin{gathered} -0.044^{* * *} \\ (0.006) \end{gathered}$ | $\begin{gathered} -0.036^{* * *} \\ (0.006) \end{gathered}$ | $\begin{gathered} -0.024^{* * *} \\ (0.007) \end{gathered}$ |
| SEZ*Firm's Share |  |  |  | $\begin{aligned} & 0.095^{* *} \\ & (0.039) \end{aligned}$ |  |  |  | $\begin{aligned} & 0.073^{* *} \\ & (0.035) \end{aligned}$ |
| SEZ*Region's Share |  |  |  | $\begin{aligned} & -0.018^{*} \\ & (0.010) \end{aligned}$ |  |  |  | $\begin{gathered} -0.021^{* *} \\ (0.010) \end{gathered}$ |
| Observations | 1346860 | 1346860 | 1346860 | 1105162 | 1346860 | 1346860 | 1346860 | 1105162 |
| Overall $R^{2}$ | 0.028 | 0.027 | 0.028 | 0.026 | 0.028 | 0.027 | 0.027 | 0.026 |
| Panel B: Dependent Variable $=\mu_{\text {nit }} /\left(\mu_{\text {nit }}-1\right)$ (full sample) |  |  |  |  |  |  |  |  |
| Firm's Share | $\begin{gathered} 225.521 \\ (200.154) \end{gathered}$ |  | $\begin{gathered} 195.421 \\ (204.416) \end{gathered}$ | $\begin{gathered} 388.275 \\ (318.785) \end{gathered}$ | $\begin{gathered} 469.783 \\ (474.848) \end{gathered}$ |  | $\begin{gathered} 439.703 \\ (480.277) \end{gathered}$ | $\begin{gathered} 800.015 \\ (772.947) \end{gathered}$ |
| Region's Share |  | $\begin{gathered} 53.610 \\ (33.033) \end{gathered}$ | $\begin{gathered} 27.789 \\ (25.908) \end{gathered}$ | $\begin{gathered} 18.889 \\ (31.765) \end{gathered}$ |  | $\begin{aligned} & 109.975 \\ & (88.601) \end{aligned}$ | $\begin{gathered} 28.807 \\ (33.949) \end{gathered}$ | $\begin{gathered} 15.500 \\ (41.299) \end{gathered}$ |
| SEZ*Firm's Share |  |  |  | $\begin{aligned} & -369.619 \\ & (251.815) \end{aligned}$ |  |  |  | $\begin{aligned} & -694.560 \\ & (525.775) \end{aligned}$ |
| SEZ*Region's Share |  |  |  | $\begin{gathered} 28.451 \\ (26.538) \end{gathered}$ |  |  |  | $\begin{gathered} 38.929 \\ (35.645) \end{gathered}$ |
| Observations | 1346860 | 1346860 | 1346860 | 1105162 | 1346860 | 1346860 | 1346860 | 1105162 |
| Overall $R^{2}$ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Panel C: Dependent Variable $=\mu_{n i t} /\left(\mu_{n i t}-1\right)\left(\right.$ drop $\left.\mu_{n i t}<1.06\right)$ |  |  |  |  |  |  |  |  |
| Firm's Share | $\begin{gathered} \hline-3.229^{* * *} \\ (0.561) \end{gathered}$ |  | $\begin{gathered} -1.985^{* * *} \\ (0.594) \end{gathered}$ | $\begin{gathered} -3.783^{* * *} \\ (0.745) \end{gathered}$ | $\begin{gathered} \hline-2.471^{* * *} \\ (0.464) \end{gathered}$ |  | $\begin{gathered} -1.311^{* * *} \\ (0.500) \end{gathered}$ | $\begin{gathered} -2.488^{* * *} \\ (0.692) \end{gathered}$ |
| Region's Share |  | $\begin{gathered} -1.404^{* * *} \\ (0.170) \end{gathered}$ | $\begin{gathered} -1.140^{* * *} \\ (0.183) \end{gathered}$ | $\begin{gathered} -1.051^{* * *} \\ (0.220) \end{gathered}$ |  | $\begin{gathered} -1.339^{* * *} \\ (0.161) \end{gathered}$ | $\begin{gathered} -1.103^{* * *} \\ (0.175) \end{gathered}$ | $\begin{gathered} -0.932^{* * *} \\ (0.208) \end{gathered}$ |
| SEZ*Firm's Share |  |  |  | $\begin{gathered} 3.248^{* * *} \\ (0.964) \end{gathered}$ |  |  |  | $\begin{gathered} 2.545^{* * *} \\ (0.905) \end{gathered}$ |
| SEZ*Region's Share |  |  |  | $\begin{aligned} & -0.272 \\ & (0.259) \end{aligned}$ |  |  |  | $\begin{aligned} & -0.372 \\ & (0.251) \end{aligned}$ |
| Observations | 1228255 | 1228255 | 1228255 | 1006748 | 1228255 | 1228255 | 1228255 | 1006748 |
| Overall $R^{2}$ | 0.010 | 0.009 | 0.009 | 0.009 | 0.010 | 0.009 | 0.009 | 0.009 |
| Panel D: Dependent Variable $=\log (\mathrm{mu})_{\text {nit }}$ |  |  |  |  |  |  |  |  |
| Firm's Share | $\begin{gathered} \hline 0.174^{* * *} \\ (0.027) \end{gathered}$ |  | $\begin{gathered} 0.118^{* * *} \\ (0.028) \end{gathered}$ | $\begin{gathered} \hline 0.193^{* * *} \\ (0.035) \end{gathered}$ | $\begin{gathered} 0.099^{* * *} \\ (0.022) \end{gathered}$ |  | $\begin{aligned} & \hline 0.051^{* *} \\ & (0.024) \end{aligned}$ | $\begin{gathered} 0.102^{* * *} \\ (0.030) \end{gathered}$ |
| Region's Share |  | $\begin{gathered} 0.067^{* * *} \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.052^{* * *} \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.035^{* * *} \\ (0.009) \end{gathered}$ |  | $\begin{gathered} 0.055^{* * *} \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.046^{* * *} \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.028^{* * *} \\ (0.009) \end{gathered}$ |
| SEZ*Firm's Share |  |  |  | $\begin{aligned} & -0.103^{*} \\ & (0.053) \end{aligned}$ |  |  |  | $\begin{aligned} & -0.085^{*} \\ & (0.047) \end{aligned}$ |
| SEZ*Region's Share |  |  |  | $\begin{aligned} & 0.034^{* *} \\ & (0.014) \end{aligned}$ |  |  |  | $\begin{gathered} 0.035^{* * *} \\ (0.013) \end{gathered}$ |
| Observations | 1346860 | 1346860 | 1346860 | 1105162 | 1346860 | 1346860 | 1346860 | 1105162 |
| Overall $R^{2}$ | 0.028 | 0.027 | 0.028 | 0.027 | 0.028 | 0.027 | 0.027 | 0.026 |
| All Panels |  |  |  |  |  |  |  |  |
| Year FEs | YES | YES | YES | YES | YES | YES | YES | YES |
| Firm FEs | YES | YES | YES | YES | YES | YES | YES | YES |

Notes: Robust standard errors clustered at firm level in parentheses. Significance: ${ }^{* * *}: 1 \%,{ }^{* *}: 5 \%,{ }^{*}: 10 \%$. Regions are defined at county level. Specifications 1-4 are weighted regressions; specifications 5-8 are unweighted regressions. All regressions include a constant term and SEZ dummy.


[^0]:    First-Stage Instruments:
    Weak Instrument ( Prob > F )
    Notes: Robust standard errors clustered at firm level in parentheses. Significance: ${ }^{* * *}: 1 \%,{ }^{* *}: 5 \%,{ }^{*}: 10 \%$. Regions are defined at various aggregation ev, et al. 2014). All regressions include a constant term.

