# CORRELATED DEFAULT RISKS AND BANK REGULATIONS

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## ABSTRACT

Bank regulatory design relies critically on bank risk modeling. Traditionally, the bank's aggregate value is assumed to obey an exogenously specified process, (e.g., a lognormal diffusion). We demonstrate that this assumption is generally invalid given the truncated and correlated payoff structure of individual bank loans. Instead, the bank's aggregate terminal payoff is significantly (left) fat tailed. This skewness remains even at the limit when the bank holds an infinitely large number of correlated loans in its portfolio. Only if the bank holds a large number of uncorrelated loans (a strong assumption) does such skewness disappear. The terminal distribution is then approximately normal (not lognormal) at the limit. By ignoring skewness in bank payoffs, deposit insurance premia and capital requirements have traditionally been significantly mis-calculated.

# 1 Introduction

Modeling bank risk is an issue of keen regulatory importance. If solvency regulations (such as deposit insurance and capital standards) do not fully adjust for the bank's asset risk, the bank would have a moral hazard incentive to increase asset risk to increase the regulator's (and ultimately the taxpayer's) loss burden.

The bank's asset portfolio is generally viewed in an "aggregated" sense in the extant literature. For instance, in a seminal paper Merton (1977) noted that the deposit insurance liability could be viewed as a put option on the bank's aggregate portfolio value with a strike price equal to the face value of the bank's deposit debt. In order to estimate the value of the deposit insurance put analytically, Merton (1977) assumed that the bank's aggregate value followed a Geometric Brownian Motion (GBM). Following Merton (1977), numerous papers have considered more refined option-theoretic approaches to pricing deposit insurance.[1] However, the majority of these papers begin with a fundamental assumption, viz., the bank's *aggregate* asset portfolio value obeys an assumed stochastic process. Such an "aggregated" approach "almost totally suppresses the question of bank portfolio composition" [Flannery (1989)]. That is, the traditional approach ignores the fact that a bank's portfolio is made up of loans to individual (corporate) borrowers with different levels of asset risk and leverage.

Two notable exceptions in this regard are papers by Dermine and Lajeri (2001) [henceforth DL] and Flannery (1989). DL consider the *borrower's* asset dynamics as the primitive risk in the economy, and highlight the truncated nature of a loan's payoff on deposit insurance premia. However, DL only consider the case of a single borrower. As a result, in DL's framework, as in the standard Merton (1977) option-theoretic context, bank shareholders are indifferent among various loan compositions so long as the portfolio attains a specific level of overall portfolio risk. In contrast, Flannery (1989) considers "the properties of individual assets (loans) *within the (bank) portfolio.*" Flannery's (1989) analysis highlights the ambiguous impact that increased

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[1] See for example, Dermine and Lajeri (2001), Duan and Yu (1999), Duan, Moreau and Sealey (1992), Chen and Mazumdar (1994), Ritchken, Thomson, DeGennaro, Pennacchi (1987), Ronn and Verma (1986), Marcus and Shaked (1984), and Merton (1978). The FDIC provides an annotated bibliography of more than 700 books and research papers written on the subject during the 1989-1999 period [FDIC (2000)].

individual loan risks has on the value of a bank's deposit insurance coverage.[2] However, unlike DL, Flannery (1989) does not analyze the truncated nature of loan payoffs and the manner in which the *borrower's* asset volatility and leverage could affect the individual loan's risk.

Our paper may be viewed as complementary to these studies. Like Flannery (1989) we too consider the bank's portfolio as comprising of several individual loans. Moreover, like DL, we model the individual borrowers' asset volatilities as the primitive risks in the system and explicitly considered the truncated payoff on bank loans. Thus, as in DL, loan risk stems from the underlying borrower's volatility and leverage in our "dis-aggregated" framework. However, unlike the extant literature, our model explicitly incorporates the effects of diversification and correlated individual loan default risks on the bank's aggregate payoff distribution.

Although such a "dis-aggregated" perspective of the bank's loan portfolio is conceptually more accurate, its practical relevance may appear to be limited at first glance. If the aggregate bank payoff (value) at maturity is approximately lognormal, the distinction between the traditional and the dis-aggregated views of the bank portfolio would be moot for practical purposes. We empirically examine the magnitude of the approximation error inherent in the aggregate approach by modeling a bank's portfolio as a portfolio of loans to a number of corporate borrowers, and assuming that the asset values of these individual corporate borrowers, rather than the aggregate value of the bank's portfolio, are lognormally distributed.

Our "dis-aggregated" model highlights two aspects of the bank portfolio. First, individual loans' terminal payoffs are not lognormally distributed since loan contracts have truncated payoffs. At a portfolio level, we demonstrate that for a bank with relatively few borrowers, the truncated nature of loan payoffs results in a significantly (left) fat-tailed distribution. Such skewness persists even if the bank is assumed to hold a diversified portfolio with a large number of loans that are positively correlated to some degree.[3]

We demonstrate through simulations that skewness in bank portfolio payoffs makes a very

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[2] In Flannery's (1989) analysis, greater individual asset risk raises the option's value directly, but it also reduces option value by lowering the strike price. The latter effect occurs because the "bank's permissible leverage (which also increases option value) is negatively related to asset risk" and declines with "the level of low-quality loans detected in its portfolio."

[3] Correlated loan default risks arise due to the correlated nature of borrowers' asset returns.

significant difference in empirical estimates of actuarially fair deposit insurance premia and capital standards. To compensate for the significantly greater downside risk associated with a fat left tailed distribution, the deposit insurance premium must be several times larger than estimates derived under the aggregated approach.

The current risk-based capital requirement system, as well as the new one being proposed under the Basel II Accord, is a "portfolio-invariant" one, *i.e.,* a bank's overall capital requirement is calculated based on its individual assets' risks rather than the bank's portfolio risk. Therefore, the level of equity capital that a bank is required to hold does not depend on the correlation across individual loans. However, we demonstrate that even if the bank held a finely-grained (or diversified) portfolio, a portfolio-invariant capital standard would significantly mis-calculate the level of equity capital the bank should maintain, given correlated borrower default risks.

The rest of this paper is organized as follows. In Section 2, we develop our dis-aggregated model of bank portfolio risk. In Section 3 we compare deposit insurance premia estimates derived under the traditional (aggregated) approach to those derived under our dis-aggregated approach. In Section 4 we analyze the capital adequacy implications of our model in light of the Internal Ratings Based (IRB) approach outlined in the recent Basel II Accord proposal. Section 5 concludes. Details for generating correlated normal random variables are given in the Appendix.

## 2 The Model

We consider an economy in a continuous time framework that comprises of corporate borrowers, a profit-maximizing bank with insured deposits and a deposit insurance agency.

## 2.1 The corporate borrowers

There are $n$ borrowing firms in the economy. We assume that each borrower's future asset value, $A_{it}$, follows GBM as described below:

$$\frac{dA_{it}}{A_{it}} = \mu_i dt + \sigma_i dW_{it}, \quad i = 1, 2, \ldots, n, \tag{1}$$

where $W_{it}$ is a standard Wiener process. The drift $\mu_i$ and asset volatility $\sigma_i$ are assumed to be constants. The instantaneous correlation coefficient, $\rho_{ij}$, between $W_{it}$ and $W_{jt}$ and the riskfree interest rate, $r$, are also assumed to be constants.

We assume that corporate debt is funded through bank loans only, all of which mature simultaneously at date $T$ and require no interim interest payments. The market value of the bank's loan to firm $i$ at time $t < T$ (before maturity), $L_{it}$, is shown below:

$$L_{it} = F_i e^{-r(T-t)} - P_{it}(A_{it}, F_i, r, \sigma_i, T-t), \tag{2}$$

where $F_i$ denotes the promised face value of loan $i$ and $P_{it}(\cdot)$ is a put option on the borrower's asset $A_i$ with a strike price of $F_i$. This put represents the borrower's option to default on the loan. In our modeling framework $P_{it}(\cdot)$ is given by the Black-Scholes formula,

$$P_{it}(A_{it}, F_i, r, \sigma_i, T-t) = F_i e^{-r(T-t)} N(-y_i + \sigma_i \sqrt{T-t}) - A_{it} N(-y_{it}), \tag{3}$$

where $N(\cdot)$ represents the cumulative standard normal distribution, and

$$y_{it} = \frac{\ln[A_{i0}/(F_i e^{-r(T-t)})]}{\sigma_i \sqrt{T-t}} + \frac{\sigma_i}{2}\sqrt{T-t}. \tag{4}$$

## 2.2 The bank and the deposit insurer

The bank's asset portfolio consists of $n$ loans, $L_{1t}$, $L_{2t}$, ..., $L_{nt}$.[4] The current market value of the bank loan portfolio, $L_t$, may be written as the sum of its individual loans' values:

$$L_t \equiv \sum_{i=1}^{n} L_{it} = \sum_{i=1}^{n} \left( F_i e^{-r(T-t)} - P_{it}(A_{it}, F_i, r, \sigma_i, T-t) \right). \tag{5}$$

Following the standard option-theoretic deposit insurance pricing literature, we assume that the bank funds its assets with insured deposits and equity and that the insurer examines the bank at date $T$, which coincides with the maturity of current loans. Since deposits are fully insured, deposits earn the riskfree rate of return. We denote the promised future value of bank deposits (including the riskfree interest) at date $T$ by $B_T$. If the bank's deposits were uninsured then their current market value would be:

$$D_t = B_T e^{-r(T-t)} - Q_t(\{A_{it}\}_{i=1}^{n}, \{F_i\}_{i=1}^{n}, \{\sigma_i\}_{i=1}^{n}, B_T, r, T-t), \tag{6}$$

where $Q_t(\cdot)$ is a European put option at time $t$ with maturity date $T$, written on the bank's aggregate assets (or portfolio) given by (5).[5] Note that $Q(\cdot)$ depends on the individual borrowing firms' current asset values, loan face values and asset volatilities in our dis-aggregated approach. The put's exercise price is the promised future value of the bank's total deposits, $B_T$. However, given deposit insurance, the value of risk-free deposits at $t = 0$ is simply $B_T e^{-rT}$. The European portfolio put, $Q(\cdot)$ denotes the deposit insurer's liability or the actuarially fair deposit insurance premium.

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[4] We ignore bank reserves in our model. However, including reserves as a part of the bank's portfolio would not qualitatively change our results.

[5] For notational convenience, later on when we refer to the initial ($t = 0$) value of $Q_0(\cdot)$, we will drop the subscript and simply refer to it as $Q(\cdot)$.

# 3 Empirical Implications for Deposit Insurance Pricing under the aggregated/Dis-aggregated Approaches

The remainder of this paper focuses on the empirical implications of our dis-aggregated approach for regulatory design. In this section we consider our model's implications in assessing actuarially fair deposit insurance premia. In the following section we consider its implications in determining capital adequacy standards.

It is well-recognized that deposit insurance is currently severely mis-priced (*i.e.* not risk-adjusted).[6] The current deposit insurance scheme was enacted through the FDIC Improvement Act in 1991 in the wake of the banking and savings and loans' crises in the 1980s. The scheme attempts to adjust for differential risk across banks by offering a premium structure that ranges from 0 to 27 cents per $100 of deposits for the riskiest banks. Banks are assigned a (fixed) premium level based on two criteria: capital levels and supervisory ratings. Notably, however, if the FDIC's reserves are adequate (as is currently the case) then the FDIC "generally may not charge premia to well-capitalized institutions with CAMELS ratings of 1 or 2" [FDIC (2001)]. As a result, currently *92% of all insured institutions are charged no deposit insurance premia at all* "and the more than 900 banks that were chartered within the last five years have never paid any premia."[7] Such free deposit insurance implies that most banks receive a deposit insurance subsidy. We now examine the degree to which deposit insurance mis-pricing may stem from an aggregated approach to bank risk modeling. We do so by comparing the value of the deposit insurance put option, $Q(\cdot)$, derived using our dis-aggregated approach [see equation (6)] to the traditional deposit insurance estimate, $P(\cdot)$, at $t = 0$.

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[6] See Alan Greenspan's April 2002 testimony to the U.S. Senate Committee on Banking, Housing and Urban Affairs (Greenspan, 2002, p.1).

[7] See FDIC (2001).

## 3.1 Deposit insurance pricing under the aggregated approach

If the bank's aggregate loan portfolio is assumed to follow a GBM then the traditional deposit insurance value, $P(\cdot)$, could be directly estimated using the Black-Scholes put formula given two parametric inputs: 1) the initial value of the bank's aggregate assets, $L_0$; and 2) the bank's aggregate asset volatility, $\sigma_{L0}$. In our modeling framework, $L_0$, given in equation (5), is expressed simply as the sum of the values of all individual loans. The bank's aggregate asset volatility could be estimated by in various ways.

One commonly used method is to de-lever the bank's (observable) stock return volatility, given the bank's capital structure.[8] Such an aggregate asset volatility estimate would implicitly capture the effects of diversification and correlation across loans, as discussed earlier. In addition to deriving implied asset volatilities from equity volatilities, Nikolova (2004) considers two alternative methodologies that incorporate observed debt prices instead of (or in addition to) equity prices. Nikolova (2004) finds that banks' implied asset volatility estimates varies considerably depending on the estimation technique used. Nikolova (2004) estimates that the median (annualized) bank asset volatility has increased over time and was in the range of 3% to 11% during the 1996-September 1999 period. These estimates are consistent with our own which are mostly in the 3% to 8% range, and were derived as discussed below.[9]

In our framework, the primitive risk in the economy is each individual borrower's asset volatility, which in turn influences the volatility of individual loans and ultimately the bank's overall loan portfolio (aggregate asset) volatility. Recall that equation (2) provides the value of each bank loan, $L_{it}$. Let $P_{it} \equiv P_{it}(A_{it}, F_i, r, \sigma_i, T-t)$ for notational convenience, we get:

$$\frac{dL_{it}}{L_{it}} = \frac{F_i e^{-r(T-t)}}{L_{it}} r dt - \frac{P_{it}}{L_{it}} \frac{dP_{it}}{P_{it}}. \tag{7}$$

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[8] For a discussion of this approach see Duan, Moreau and Sealey (1992), Pennacchi (1987), Ronn and Verma (1986) and Marcus and Shaked (1984).

[9] Note that regardless of the method used to estimate aggregate asset volatility, any deposit insurance estimate derived using the volatility estimate as an input in the Black-Scholes model would be flawed since the aggregate asset's true distribution is not lognormal.

The standard deviation of $dP_{it}/P_{it}$ is given by

$$SD\left(\frac{dP_{it}}{P_{it}}\right) = SD\left(\frac{dP_{it}}{dA_{it}}\frac{dA_{it}}{A_{it}}\frac{A_{it}}{P_{it}}\right) = \left|\frac{\partial P_{it}}{\partial A_{it}}\right|\frac{A_{it}}{P_{it}}SD\left(\frac{dA_{it}}{A_{it}}\right) = N(-y_{it})\frac{A_{it}}{P_{it}}\sigma_i, \tag{8}$$

Hence, the standard deviation of $dL_{it}/L_{it}$ is:

$$\sigma_{L_{it}} = SD\left(\frac{dL_{it}}{L_{it}}\right) = \frac{P_{it}}{L_{it}}SD\left(\frac{dP_{it}}{P_{it}}\right) = \frac{A_{it}}{L_{it}}N(-y_{it})\sigma_i. \tag{9}$$

Finally, the bank's aggregate portfolio volatility is:

$$\sigma_{Lt} = \left(\sum_i\sum_j x_{it}x_{jt}\sigma_{L_{it}}\sigma_{L_{jt}}\rho_{ij}\right)^{0.5}, \tag{10}$$

where $x_{it} = L_{it}/L_t$ is the fraction that the market value of loan $i$ represents of the total market value of the bank's portfolio and $\rho_{ij}$ is the correlation coefficient between asset returns of firm $i$ and firm $j$. Note that the portfolio volatility in (10) varies over time, $t$.

The aggregate parameter values at the initial time 0, $L_0$ and $\sigma_{L0}$, can then be used as inputs in the Black-Scholes put formula to derive the deposit insurance premium under the aggregated approach, $P_0(L_0, B_T, r, \sigma_{L0}, T)$.[10]

## 3.2 Deposit insurance pricing under the dis-aggregated approach

To price the deposit insurance value under the dis-aggregated approach, $Q(\cdot)$, we start by noting that the bank's asset value at time $T$, $V_B(T)$, consists of the payments received on individual loans, which are all assumed to mature at date $T$. That is,

$$\begin{aligned} V_B(T) &= \sum_{i=1}^{n}\min(F_i, A_{iT}) = \sum_{i=1}^{n}\left(F_i 1(A_{iT} > F_i) + A_{iT}1(A_{iT} < F_i)\right) \\ &= \sum_{i=1}^{n}F_i - \sum_{i=1}^{n}(F_i - A_{iT})1(A_{iT} < F_i), \end{aligned} \tag{11}$$

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[10] For notational simplicity, we will refer to $P_0(\cdot)$ as $P(\cdot)$.

where $1(\cdot)$ is the indicator function. Thus, equation (11) indicates that the bank's total receipts at date $T$ equal the total promised payment on all loans made less the total loss associated with loans to those borrowers that are insolvent at date $T$. The latter value is simply the sum of the payoffs on the default puts that were in-the-money at date $T$ [see equation (5)].

The strike price of the insurance put, $B_T$, is the face value of bank deposits at $T$. Therefore, the insurance value at maturity is $(B_T - V_B(T))^+$. Risk-neutral pricing implies that the put's current value (which equals the actuarially fair deposit insurance premium) is given by:

$$Q(\cdot) = e^{-rT} E_0^q[(B_T - V_B(T))^+], \tag{12}$$

where $q$ denotes the risk-neutral measure under which

$$A_{iT} = A_{i0} e^{(r-\sigma_i^2/2)T + \sigma_i W_{iT}}. \tag{13}$$

Therefore, (12) becomes

$$Q(\cdot) = e^{-rT} E_0^q \left[ \left( B_T - \sum_{i=1}^{n} F_i + \sum_{i=1}^{n} (F_i - A_{i0} e^{(r-\sigma_i^2/2)T + \sigma_i W_{iT}}) 1(A_{i0} e^{(r-\sigma_i^2/2)T + \sigma_i W_{iT}} < F_i) \right)^+ \right]. \tag{14}$$

The expectation is with respect to the random variables $W_{iT}$, $i = 1, 2, \cdots, n$.

Since $W_{it}$ is a standard Wiener process, for a fixed time $t = T$, $W_{iT}$ is normally distributed with mean 0 and variance $T$. Thus, we can let $W_{iT} = \sqrt{T}\varphi_i$ where $\varphi_i$ is a standard normal random variable with mean 0 and variance 1. Since the correlation between $W_{iT}$ and $W_{jT}$ is $\rho_{ij}$, the correlation between $\varphi_i$ and $\varphi_j$ is also $\rho_{ij}$. Using the correlated standard normal random variables $\varphi_i$, $i = 1, 2, \cdots, n$, we can rewrite (14) as,

$$Q(\cdot) = e^{-rT} E \left[ \left( B_T - \sum_{i=1}^{n} F_i + \sum_{i=1}^{n} (F_i - A_{i0} e^{(r-\sigma_i^2/2)T + \sigma_i \sqrt{T}\varphi_i}) 1(A_{i0} e^{(r-\sigma_i^2/2)T + \sigma_i \sqrt{T}\varphi_i} < F_i) \right)^+ \right]. \tag{15}$$

The expectation in (15) is with respect to the correlated normal random variables, $\varphi_i$, $i = 1, 2, \cdots, n$. Although it is not possible to obtain (15) in closed form, numerical values can be obtained using Monte Carlo simulations. One draw of $n$ correlated normal random variables yields one possible option value. Averaging over a large number of values obtained through multiple draws (or simulations) yields the option value, $Q(\cdot)$. Details regarding the generation of correlated normal random variables are provided in the Appendix.

## 3.3 Comparison of deposit insurance estimates under the alternative approaches

We next compare the traditional deposit insurance put estimate (per \$100 of insured deposits), $P(\cdot)/B_0\,(\%)$, to that under our dis-aggregated approach, $Q(\cdot)/B_0\,(\%)$. The base case parameters for the simulation exercises are as follows: Riskfree rate, $r = 5\%$; Individual borrower $i$'s current asset value, $A_{i0} = 10$; Borrower $i$'s level of bank debt (in terms of face value at maturity), $F_i = 9$; Loan maturity (in years), $T = 1$; Number of loans in bank portfolio, $n = 10$; Correlation across (pairs) of borrowers' returns, $\rho_{ij} = 0.5$; Individual borrower asset volatility, $\sigma_i = 0.3$, $\forall\, i$;[11] Face value of bank deposits, $B_T = 0.9L$.[12]

Table 1 indicates that, in contrast to our dis-aggregated approach's estimates, the traditional deposit insurance estimate significantly underprices the insurer's liability and in some cases estimates it at close to zero. The first row in Table 1 reports the base case results. We make two observations in this context. First, even though the individual borrower firms' asset volatilities are assumed to be 30%, the volatility of a loan to such a borrower, $(\sigma_{L_i})$, is much smaller at 9.42%, given the loan's senior claim on the borrower's assets. The overall bank loan portfolio (which consists of 10 loans) also has a relatively low volatility $(\sigma_L)$ of 6.98%, which is well within the range suggested by extant empirical studies [see Nikolova (2004)].[13] Sec-

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[11] We consider changes in the individual borrower's asset volatility over the range of 20% to 40% in our simulations. This range is consistent with extant empirical estimates of asset volatility. For instance, Ju, Parrino, Poteshman and Weisbach (2003) estimated that the median asset volatility was 38% for a large sample of over 2,000 firms. Nikolova (2003) estimated industrial firms' assets volatilities for a sample of 19,020 quarterly observations. The median estimates derived using four alternative methodologies ranged from 14.57% to 32.86%.

[12] Thus, the bank's market value of equity is over 10% of its current asset value because the market value of the loan, $L_i$ is smaller than $F_i$ which is 9. This parametric specification is in the spirit of current FDICIA regulations that mandate that "well-capitalized" institutions maintain a 10% risk-based capital ratio.

[13] Depending on the estimation methodology used, the *maximum* bank implied asset volatility estimate ranges from 19.48% to 20.55%, in Nikolova's (2004) sample of 2,060 firm-quarter observations over the period 1986-1999 [see Nikolova (2004), Table 1].

ond, the values of $P(\cdot)/B_0(\%)$ and $Q(\cdot)/B_0(\%)$ (per \$100 of insured deposits) are significantly different.[14] The traditional deposit insurance estimate, $P(\cdot)/B_0(\%)$, is approximately 3 cents per \$100 of deposits. In contrast, the premium estimate under the dis-aggregated approach, $Q(\cdot)/B_0(\%)$, is 56 cents per \$100 of deposits.

The traditional approach yields a significantly lower deposit insurance premium estimate for two reasons. First, the loan portfolio volatility estimate that is based on the underlying individual loan values ($\sigma_{L0}$, column 2) is very small (6.985%) in the base case. Second, the Black-Scholes put option is initially 10% out-of-the-money. As a result, under the standard Black-Scholes option pricing framework, the put option's value is negligible, when the aggregate loan portfolio is assumed to be the underlying stochastic variable. The wide disparity between the traditional approach's deposit insurance premium estimates and those obtained under our dis-aggregated approach indicates that the traditional approach provides a poor approximation for the insurer's true contingent liability. In other words, our results indicate that the future bank asset value distribution has a much fatter left tail than the lognormal distribution that is assumed traditionally.[15] The difference between the traditional and dis-aggregated approaches is further highlighted through changes in parameter values one at a time (as shown in column 1).

The last column in Table 1 represents the traditional deposit insurance estimate as a percent of the dis-aggregated insurance premium estimate, $Q(\cdot)$. That is, $\phi$ is defined as $\phi = 100\,(P(\cdot)/Q(\cdot))$. The magnitude of the under-pricing inherent in the aggregated approach is apparent by reviewing the last column of Table 1. This column indicates that the traditional deposit insurance estimate never exceeds more that 35% of the more accurate estimate derived under the dis-aggregated approach.

Moreover, both deposit insurance premia estimates increase with an *increase* in the loan maturity, individual borrower asset volatility, correlation across borrower returns, the bank's leverage ratio or a *decrease* in the riskfree rate. Although $\phi$ increases with such parametric

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[14] $B_0 = B_T e^{-rT}$ is the value of deposits at time zero. Remember that $B_T$ is the deposits due at time $T$.

[15] We discuss such difference in distributional forms later in subsection 3.5.

changes (or the under-pricing bias in percentage terms declines), the disparity between $P(\cdot)$ and $Q(\cdot)$ widens in absolute terms. This is troubling from a policy perspective since the fair pricing of deposit insurance is especially important as the borrowers' risk, leverage and prospects of correlated defaults increase.

To highlight this point let us consider the effect of an increase in individual borrowers' asset volatilities from the base case of 30% to 40%. As a result, the individual loan volatilities increase from 9.42% to 14.45% and the overall loan portfolio volatility increases from 6.98% to 10.71%. The loan portfolio value declines from \$80.30 in the base case to \$77.02. The Merton-style deposit insurance estimate (using the Black-Scholes model) increases to 38 cents per \$100 of deposits. In contrast, the value of the dis-aggregated deposit insurance premium rises to \$1.48 per \$100 of deposits. The disparity between the insurance premia thus more than doubles to \$1.10 per \$100 of deposits, from about \$0.53 per \$100 of deposits in the base case.

## 3.4 Deposit insurance pricing and the number of bank loans held in a portfolio

In this sub-section we consider whether our earlier results are qualitatively affected if we increase the number of loans in the bank's portfolio from 10 to 100. The new results with 100 loans are shown in Tables 2.[16] Tables 1 and 2 are comparable on a row by row basis. Table 2 highlights three main results. First, a row-by-row comparison of Table 2 to Table 1 indicates that as the number of loans held in the bank's portfolio increases from 10 to 100, the deposit insurance premium estimates under either the aggregated or dis-aggregated approaches decline marginally [see columns 7 and 9, respectively).[17]

Second, comparing the last columns of Tables 1 and 2 indicates that the underpricing bias

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[16] To maintain the same aggregate bank loan portfolio market value, the initial asset values and loan face values for each borrowing firm is proportionately reduced by a factor of 10 in Table 2. The other parameter values are kept the same.

[17] Moreover, the decline in the premium estimate is negligible when we change the number of loans from 100 to 1,000, indicating that the bank loan portfolio is almost fully diversified with 100 loans. Results using 1,000 loans are not reported here to conserve space, but are qualitatively the same and available upon request.

inherent in the aggregated approach generally increases (or $\phi$ declines) as the number of loans in the bank's portfolio increases. This result suggests that even for a well-diversified portfolio, the skewness associated with the bank's aggregate terminal payoff remains significant.

Third, a notable exception to the second result is the uncorrelated default risk case ($\rho_{ij} = 0$). In this instance, as the number of loans increases, both the aggregated and dis-aggregated approach yields a deposit insurance premium estimate close to zero. This follows from the parametric specification that the bank's total deposits at maturity, $B_T$ is smaller than the bank's current aggregate loan portfolio value, $L_0$ and the fact that the bank's aggregate portfolio risk approaches zero if the bank holds a large number of pair-wise *uncorrelated* loans. In such a scenario, the bank's portfolio value almost certainly exceeds the outstanding level of deposits, $B_T$, at maturity and the prospect of bank failure is negligible. Accordingly, the deposit insurance premium estimated under either approach is then approximately zero.

In short, our analysis reveals that the truncated nature of loan payoffs leads to a skewed terminal payoff distribution for the bank's portfolio as a whole. This skewness persists even at the limit when the bank holds an infinitely large number of loans, as long as these loans are partially correlated. Only in the highly unrealistic case when all loans are assumed to be uncorrelated would such skewness in the bank's aggregate payoff diminish as the bank held an increasingly diversified portfolio. The distribution of the bank's terminal payoff would instead look approximately normal, as the Central Limit Theorem dictates. To demonstrate this we conduct a series of Monte Carlo simulations in the next subsection to obtain the terminal distribution of the bank's loan portfolio value, which is given by (11).

## 3.5 The relationship between the bank's terminal cash flow distribution and the number of loans

The distribution of the sum of a series of random variables is in general difficult to represent formally.[18] Under strong conditions, for example, for a set of identically independent

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[18] We thank Dilip Madan for helpful conversations on this topic. For more details, see, e.g. Durrett (1991).

distributed (i.i.d.) random variables, the well-known Central Limit Theorem holds. Under more general conditions, no formal conclusion can be drawn. Therefore, to better understand how changing the number of loans held by the bank affects the bank's terminal cash flow distribution ("distribution") and hence deposit insurance pricing, we plot Figures 1-6. In these figures we compare the "true" (simulated) distribution under our dis-aggregated approach to the normal distribution (the converging distribution if the Central Limit Theorem applied), and the lognormal distribution (which bank asset values are assumed to follow under the traditional approach).

In Figures 1-4, the parameter values correspond to the base case values in Table 1 except for the number of loans (non-independent borrowing firms).[19] Thus, in figures 1-4, the individual borrowers' asset returns are assumed to be correlated, with $\rho_{ij} = 0.5$. The parameter values are the same for Figures 5-6 except that correlation coefficients are assumed to be zero (independent borrowing firms).

Panel A of each figure plots the density associated with specific bank terminal values. Note that the loan portfolio's terminal value has an upper limit at 90, which is the sum of all loans' face values. Therefore, in Panel A of Figures 1-6, the simulated distribution has zero probability *above* 90. However, note that there is a *point* mass *at* 90 which is the (joint) probability that all loans are paid in full (*i.e.,* all individual borrowers' terminal asset values are above their loan face values). In other words, the total cumulative probability under the dotted line in Panel A of each figure is not 1.[20] Panel B of Figures 1-6 represents the payoff range of the deposit insurance put option, *i.e.,* the range of bank portfolio cash values which lies below the face value of total bank deposits, $B_T = 72.27$.[21]

Figures 1-4 indicate that with correlated loans, the true distribution of the terminal loan

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[19] Remember that the initial asset value and the face value of the loan of each borrowing firm are changed accordingly so that the loan portfolio value is kept the same, in this case 80.30.

[20] Since a point mass has infinite density, it cannot be graphically represented. Another way to present the distributions is to plot the cumulative functions. In this case, there will be a jump from below to 1 at 90 for the simulated distribution. However, since the put option value is the expected value of its discounted payoffs, the density plot is more informative.

[21] Recall that in the base case of Table 1, the initial loan portfolio value (or the bank's asset value), $L_0$, is 80.30. The face value of total deposits is assumed to be 90% of the bank asset value or 72.27 ($0.9 * 80.30$).

portfolio value looks very different compared to the normal or lognormal distributions. For example, when the bank has only one loan in its portfolio, the true distribution has a long and fat left tail and a significant (simulated) point mass probability of 0.6452 at the terminal value of 90. In this one loan case, the point mass probability at 90 can also be analytically derived to be 0.6435, indicating that our simulation is quite accurate.

As the number of loans held in the bank portfolio increases, the (simulated) point mass probabilities (joint probabilities of all loans paid in full) at 90 declines from 0.2076 (in Figure 2 where $n = 10$) to 0.0800 (in Figure 3 where $n = 100$)and 0.0629 (in Figure 4 where $n = 1,000$). The intuition behind this result is straightforward: the joint probability that *all* borrowing firms will successfully pay off their loans declines as the number of loans held increases. The distributions with $n = 100$ and $n = 1,000$ are practically the same, once again indicating that the portfolio with 100 loans is almost fully diversified. The "true" distribution has a fatter tail and is distinctively different from the lognormal distribution even with $n = 1,000$.

Figures 5 and 6 shed additional light on the manner in which correlation across borrowers' returns affects the terminal distribution of the bank's cash flows and deposit insurance pricing. In Figures 5-6, all borrower returns are assumed to be uncorrelated. The true distribution's point mass probability at the terminal value of 90 is 0.0140 in Figure 5 (where $n = 10$) and close to zero in Figure 6 (where $n = 100$). It is evident from Panel A of Figures 5 and 6 that when borrower returns are uncorrelated the true distribution does approach the normal distribution, as the Central Limit Theorem predicts, as the number of bank loans increases. Furthermore, with uncorrelated borrower returns, the terminal loan portfolio value's variance becomes increasingly smaller as the number of loans increases.

Thus, our results highlight the fact that unless all borrower returns are strictly uncorrelated (which is a very strong assumption), non-diversifiable risks exist. Therefore, in general, with correlated borrower returns, the bank's (terminal) portfolio value's limiting distribution does not approach the normal or lognormal distributions, even when the number of loans is very large. As a result, deposit insurance mis-pricing under the aggregated approach remains a significant problem even for the case of well-diversified banks.

# 4 Capital Adequacy

To reduce the burden on the bank insurer, insured banks are required to maintain minimum levels of capital as well as *risk-based* capital. The current risk-based capital scheme classifies bank assets into different risk classes or "buckets" and requires the bank to hold a minimum level of capital relative to its risk-weighted asset balance. Thus, the risk-based capital scheme requires the bank to hold relatively more capital for riskier assets. However, such a bucket-based system provides banks with an incentive to undertake regulatory arbitrage. Since not all assets within a risk class are equally risky and yet are assigned the same capital requirement, banks have an incentive to move safer assets within a class off-balance sheet and retain the riskiest ones on their books. It is well recognized that as long as capital charges are not closely aligned to individual asset risk the incentive for such regulatory arbitrage would remain.

## 4.1 The internal ratings based (IRB) approach to measuring capital adequacy

The recent Basel Capital Accord (Basel II) proposal[22] suggests an Internal Ratings Based (IRB) approach of setting bank's minimum capital requirements. Such a proposal is intended to more accurately align capital requirements with the bank's intrinsic amount of credit risk exposure in a manner that is consistent with current risk management systems employed by banks.[23] Thus, IRB capital requirements would rely on a finer partitioning of assets to be based on risk attributes such as by internal borrower rating; loan type (sovereign v. corporate v. project finance); proxies such as seniority/collateral; and maturity. However, as Gordy (2003) notes, the proposed IRB approach largely remains a rating (or "bucket")-based system.

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[22] See Basel Committee on Banking Supervision, Consultative Document, Overview of The New Basel Capital Accord, issued for comments on 31 July 2003; dated April 2003. Source: http://www.bis.org/bcbs/cp3ov.pdf. [Henceforth referred to as CP3].

[23] The IRB approach includes two variants: a foundation version and an advanced version that differ primarily in terms of the inputs that are provided by the bank based on its own estimates and those that have been specified by the supervisor. In the U.S., ten or so of the largest internationally active banks would be required to use the advanced IRB approach. Other U.S. banks may compute their risk-based capital on the basis of Basel I or adopt Basel II if they wish and their regulatory supervisors concur [Kaufman (2003)].

Despite these proposed changes, the capital adequacy rules would remain *portfolio-invariant*. That is, the capital charge on a given instrument would continue to depend only on its own characteristics, and not the characteristics of the portfolio in which it is held [Gordy (2003)]. As a result, the IRB approach without further adjustments would continue to mis-calculate the level of economic capital the bank should maintain and give the bank a regulatory moral hazard incentive.

## 4.2 The dis-aggregated bank portfolio model and capital requirements

Risk-adjusted deposit insurance premia and capital requirements are complementary regulations that share the same objective, namely to protect the deposit insurer from loss. The former estimates the loss to the insurer given default by the banks' borrowers while the latter estimates the economic capital the bank should contribute to buffer the insurer from such loss. In the remainder of this section, we examine the relationship between deposit insurance pricing and capital adequacy requirements in more detail.

We derive the aggregate equity-asset (or "capital") ratio ($E_Q/L_0$) that the bank must hold to ensure that the given deposit insurance premium was actuarially fair under the dis-aggregated approach. We contrast this capital ratio to $E_P/L_0$, which denotes the appropriate capital ratio assuming the deposit insurance was priced at $P(\cdot)$ under the aggregated approach. These results are shown in Table 3.[24]

In Table 3, Panel A, the deposit insurance premium is fixed at 0.25% of the bank's total deposits. The capital requirement (which would make the 0.25% deposit insurance premium actuarially fair) is significantly lower under the "aggregated approach," (9.52%) compared to the dis-aggregated approach (20.26%) for the base case. This difference stems from the fat left-tail of the portfolio terminal cash flow distribution modeled under the dis-aggregated approach and the resulting larger insurance premium. As a result, under the aggregated

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[24] The base case parameters in Table 3 are assumed to be identical to those in Table 1. Therefore, as shown earlier in Table 1, the current market value of the bank's aggregate loan portfolio in the base case is \$80.30.

approach, the bank uses \$72.64 of deposit financing to fund its loan portfolio of \$80.30 and pays a total deposit insurance premium of 0.25% of \$72.64 (or \$0.18). In contrast, under the dis-aggregated approach, the bank uses \$64.03 of deposit financing to fund its loan portfolio of \$80.30 and pays a total deposit insurance premium of 0.25% of \$64.03 (or \$0.16).

As the rest of Table 3 indicates the difference between the capital requirements estimated under the aggregated and dis-aggregated approaches remains significant regardless of the set of parameters used for the simulation analysis.

## 4.3 Portfolio diversification and capital adequacy

The Basle Committee has recognized that portfolio-invariant capital rules may lead to a mis-calculation of the level of economic capital that the bank should hold, but the Committee opines that as long as a bank's portfolio is sufficiently diversified into several relatively small loans or "finely-grained," such mis-calculation would be small.[25] Accordingly, the Committee has suggested that no single retail exposure should exceed 0.2% of the overall regulatory retail portfolio.[26]

However, such a granularity adjustment ignores the correlated nature of loan default risks. Even at the limit, as long as loans are correlated, the bank's overall portfolio remains risky.[27] Therefore, the bank would have an incentive to make highly correlated loans to maximize its deposit insurance subsidy since under a portfolio-invariant capital requirement the bank would not be required to concomitantly maintain a higher equity-asset ratio. Table 4 illustrates the magnitude of this problem.

We start by setting the deposit insurance premium to equal 1.71%of deposits (which, can be shown to be actuarially fair under the dis-aggregated approach when $n = 10$, $\rho = 0.3$ and an aggregate capital-asset ratio of 11.43%). This is the base case shown in column 8 of

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[25] Granularity describes the extent of single borrower concentrations in the bank's portfolio.

[26] Basel Committee on Banking Supervision, April 2003, "Consultative Document: The New Basel Capital Accord, Issued for comment by 31 July 2003," paragraph 44, page 11.

[27] In contrast, the Basle Committee opines that at the limit all "idiosyncratic risk" could be diversified away. See "The Basel Committee on Banking Supervision, January 2001,'Consultative Document: The Internal Ratings-Based Approach," Chapter 8: Granularity, page 89.

Table 4. We then vary the correlation across the bank's borrowers, $\rho_{ij}$, from 0.6 in column 2, to 0.3 in column 8 to 0 in column 14. Each column of Table 4 reports the aggregate capital ratio that the bank should maintain (given other parameter estimates) for the 1.71% insurance premium to remain actuarially fair. As loan correlation increases, the bank should be required to maintain a higher equity ratio. For instance, a bank with 10 loans (with evenly-spaced borrower asset volatilities from 0% to 100%) would require a capital ratio of 11.43% if $\rho_{ij}$ was 0.3 (the base case). But this capital ratio would nearly double to 19.34% if $\rho_{ij}$ increased to 0.6. However, under a portfolio-invariant capital adequacy rule, the bank's capital requirement would remain unchanged and the bank would have an incentive to maximize its regulatory subsidy with highly correlated loans.

Even when the number of loans increases from 10 to 100, (as shown in Table 4, Panel B) the conclusions remain qualitatively unchanged as long as loans are correlated. Despite reducing the portfolio's granularity, loan correlations significantly affect the fair pricing of deposit insurance and the corresponding capital ratio that the bank should maintain. For instance, if loan correlations were 0.6 instead of 0.3, the bank would have to maintain a 22.27% equity-asset ratio instead of 11.59% for deposit insurance to remain fairly priced under the disaggregated approach. However, under a portfolio-invariant capital rule, the bank's required capital ratio would not be a function of loan correlations and hence would remain at the base value of 11.59% in our example.

A final comment regarding the capital requirement scheme, $E_Q/L\,(\%)$, shown in Table 4, is in order. $E_Q/L\,(\%)$ denotes the aggregate capital-asset ratio that the bank must maintain for the regulatory structure to be actuarially fair. Clearly, this aggregate capital requirement could be satisfied by an infinite array of asset-specific capital charges. However, the point of our preceding discussion is that such asset-specific capital charges (which, by definition, would be portfolio invariant) are irrelevant as long as the bank satisfied an aggregate capital requirement that explicitly considered the fat-tailed nature of the aggregate bank portfolio payoff at maturity.

# 5 Conclusions

Since the Great Depression, various solvency-related bank regulations have been introduced to ensure the safety and soundness of the banking system. The design of such regulations (deposit insurance and risk-based capital requirements) depends critically on the assumptions made regarding the stochastic dynamics of a bank's portfolio. Traditionally, the deposit insurer's liability has been viewed as a put option on the bank's aggregate assets, which is assumed to obey an exogenously specified process, (*e.g.* a lognormal diffusion).

We demonstrate that this assumption is generally invalid given the truncated payoff structure of individual bank loans. Instead, the bank's aggregate terminal payoff is significantly (left) fat tailed. Diversification does not necessarily eradicate such skewness. Even when the bank is assumed to hold a finely-grained portfolio consisting of numerous loans, the bank's aggregate terminal payoff remains significantly skewed to the left, as long as the loans are correlated to some degree.

While theoretically valid, does such skewness really make a practical difference in pricing deposit insurance or setting risk-based capital standards? We demonstrate through Monte Carlo simulations that our dis-aggregated approach yields significantly different deposit insurance premia and capital requirement estimates compared to the traditional aggregated approach that ignores the characteristics of individual corporate borrowers and loans. Recent policy proposals, such as the granularity adjustment suggested in Basle II, do not consider the correlated nature of individual loan default risks and hence continue to potentially miscalculate the level of economic capital that banks should hold to ensure that the regulatory system is fairly priced.

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# Appendix

The key to compute (17) using Monte Carlo simulations is to generate a vector of *correlated* standard normal random variables, $\varphi$, with correlation matrix $\rho$. The standard procedure consists of three steps.

1: generate a vector of *independent* standard normal random variables, $\tilde{\varphi}$. Many software packages have independent normal random variable generators.

2: find the eigenvector matrix and eigenvalue vector of the correlation matrix $\rho$. Standard linear algebra routines exist to find the eigenvalue and eigenvector of a real symmetric matrix. Let the eigenvector matrix be $\Omega$ and the diagonal eigenvalue matrix be $\Gamma$. Then, of course, $\rho$ has the spectrum decomposition, $\rho = \Omega\,\Gamma\,\Omega^{\mathrm{T}}$, where T denotes the transpose.

3: Let $\Lambda = \Gamma^{1/2}$ be the diagonal matrix whose elements are the square-root of the eigenvalues. Then $\varphi = \Omega\,\Lambda\,\tilde{\varphi}$ is a vector of *correlated* standard normal random variables with correlation matrix $\rho$. To check, the variance-covariance matrix $= E[\varphi\,\varphi^{\mathrm{T}}] = \Omega\,\Lambda\,E[\tilde{\varphi}\,\tilde{\varphi}^{\mathrm{T}}]\,\Lambda^{\mathrm{T}}\,\Omega^{\mathrm{T}} = \Omega\,\Lambda\,I\,\Lambda^{\mathrm{T}}\,\Omega^{\mathrm{T}} = \Omega\,\Gamma\,\Omega^{\mathrm{T}} = \rho$, where $I$ is the identity matrix. Note that we have used the relations that $E[\tilde{\varphi}\,\tilde{\varphi}^{\mathrm{T}}] = I$ because $\tilde{\varphi}$ is a vector of independent standard normal random variables and $\Lambda\,I\,\Lambda^{\mathrm{T}} = \Lambda\,\Lambda^{\mathrm{T}} = \Lambda^2 = \Gamma$ because $\Lambda$ is symmetric.

Table 1: **Deposit Insurance Premium Estimates Under the Traditional Approach, $P(\cdot)$ and the Dis-Aggregated one, $Q(\cdot)$, that Considers the Individual Loans' Default Risks**. Column 1 reports the alternative parameter values considered for the specific simulation. Columns 2-3 report the individual loan and loan portfolio standard deviation, respectively. Column 4 presents the bank's total asset value (current loan portfolio value). Column 5 indicates the face value of bank deposits which is assumed to be equal to 90% of the current loan portfolio value. Column 6 presents the traditional Merton put deposit insurance put value, $P(\cdot)$. Column 7 represents the put premium $P(\cdot)$ per \$100 of deposits at time 0 ($B_0 = B_T e^{-rT}$). Column 8 presents the deposit insurance premium $Q(\cdot)$ estimated under the disaggregated approach, using Monte Carlo with 2 million simulations. Column 9 presents $Q(\cdot)$ per \$100 of deposits at time 0. The last column reports $P(\cdot)$ as a percent of $Q(\cdot)$, i.e. $\phi = 100\,(P(\cdot)/Q(\cdot))$. The number of borrowing firms, $n$, is fixed at 10.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| | $\sigma_{L_{i0}}$ | $\sigma_{L0}$ | $L_0$ | $B_0$ | $P(\cdot)$ | $P(\cdot)/B_0(\%)$ | $Q(\cdot)$ | $Q(\cdot)/B_0(\%)$ | $\phi$ |
| Base | 0.0942 | 0.0698 | 80.30 | 72.27 | 0.0237 | 0.0344 | 0.3881 | 0.5645 | 6.0939 |
| $r = 0.09$ | 0.0813 | 0.0603 | 78.06 | 70.25 | 0.0007 | 0.0011 | 0.1694 | 0.2638 | 0.4170 |
| $r = 0.07$ | 0.0876 | 0.0650 | 79.19 | 71.27 | 0.0050 | 0.0075 | 0.2596 | 0.3906 | 1.9201 |
| $r = 0.03$ | 0.1009 | 0.0749 | 81.39 | 73.25 | 0.0796 | 0.1119 | 0.5668 | 0.7973 | 14.0349 |
| $r = 0.01$ | 0.1079 | 0.0800 | 82.46 | 74.22 | 0.2078 | 0.2828 | 0.8092 | 1.1013 | 25.6787 |
| $T = 2.0$ | 0.0989 | 0.0733 | 73.76 | 66.38 | 0.0618 | 0.1029 | 0.5984 | 0.9962 | 10.3293 |
| $T = 1.5$ | 0.0976 | 0.0724 | 76.82 | 69.14 | 0.0482 | 0.0751 | 0.5238 | 0.8166 | 9.1967 |
| $T = 0.5$ | 0.0836 | 0.0620 | 84.51 | 76.06 | 0.0015 | 0.0019 | 0.1655 | 0.2230 | 0.8520 |
| $T = 0.25$ | 0.0670 | 0.0497 | 87.14 | 78.43 | 0.0000 | 0.0000 | 0.0393 | 0.0507 | 0.0000 |
| $F_{iT} = 8.5$ | 0.0760 | 0.0564 | 77.08 | 69.37 | 0.0035 | 0.0054 | 0.2485 | 0.3766 | 1.4339 |
| $F_{iT} = 8.0$ | 0.0589 | 0.0437 | 73.54 | 66.18 | 0.0001 | 0.0002 | 0.1444 | 0.2293 | 0.0872 |
| $F_{iT} = 7.5$ | 0.0435 | 0.0323 | 69.70 | 62.73 | 0.0000 | 0.0000 | 0.0745 | 0.1249 | 0.0000 |
| $F_{iT} = 7.0$ | 0.0302 | 0.0224 | 65.60 | 59.04 | 0.0000 | 0.0000 | 0.0334 | 0.0594 | 0.0000 |
| $\sigma_i = 0.45$ | 0.1698 | 0.1259 | 75.33 | 67.80 | 0.4588 | 0.7114 | 1.3233 | 2.0520 | 34.6686 |
| $\sigma_i = 0.40$ | 0.1445 | 0.1071 | 77.02 | 69.31 | 0.2501 | 0.3794 | 0.9774 | 1.4824 | 25.5936 |
| $\sigma_i = 0.35$ | 0.1192 | 0.0884 | 78.68 | 70.81 | 0.1022 | 0.1517 | 0.6603 | 0.9802 | 15.4764 |
| $\sigma_i = 0.20$ | 0.0457 | 0.0339 | 83.30 | 74.97 | 0.0000 | 0.0000 | 0.0528 | 0.0741 | 0.0000 |
| $\rho_{ij} = 0.8$ | 0.0942 | 0.0853 | 80.30 | 72.27 | 0.0856 | 0.1245 | 0.8650 | 1.2582 | 9.8951 |
| $\rho_{ij} = 0.6$ | 0.0942 | 0.0753 | 80.30 | 72.27 | 0.0402 | 0.0585 | 0.5322 | 0.7742 | 7.5562 |
| $\rho_{ij} = 0.3$ | 0.0942 | 0.0573 | 80.30 | 72.27 | 0.0043 | 0.0063 | 0.2610 | 0.3796 | 1.6596 |
| $\rho_{ij} = 0.0$ | 0.0942 | 0.0298 | 80.30 | 72.27 | 0.0000 | 0.0000 | 0.0011 | 0.0016 | 0.0000 |
| $B_T/L_0 = 0.85$ | 0.0942 | 0.0698 | 80.30 | 68.26 | 0.0017 | 0.0025 | 0.1894 | 0.2917 | 1.6596 |
| $B_T/L_0 = 0.80$ | 0.0942 | 0.0698 | 80.30 | 64.24 | 0.0001 | 0.0001 | 0.0850 | 0.1392 | 0.0718 |
| $B_T/L_0 = 0.75$ | 0.0942 | 0.0698 | 80.30 | 60.23 | 0.0000 | 0.0000 | 0.0345 | 0.0602 | 0.0000 |
| $B_T/L_0 = 0.70$ | 0.0942 | 0.0698 | 80.30 | 56.21 | 0.0000 | 0.0000 | 0.0123 | 0.0230 | 0.0000 |

Table 2: **Deposit Insurance Premium Estimates Under the Traditional Approach, $P(\cdot)$, and the Dis-Aggregated one, $Q(\cdot)$, that Considers the Individual Loans' Default Risks**. This table is similar to Table 1, except that the number of borrowing firms, $n$, is fixed at 100. To make the numbers comparable, the size of each individual loan is reduced by a factor of 10. For example, the total loan value is still \$80.30 in the base case even though the number of loans increased from 10 to 100. $Q(\cdot)$ is estimated using 0.2 million Monte Carlo simulations.

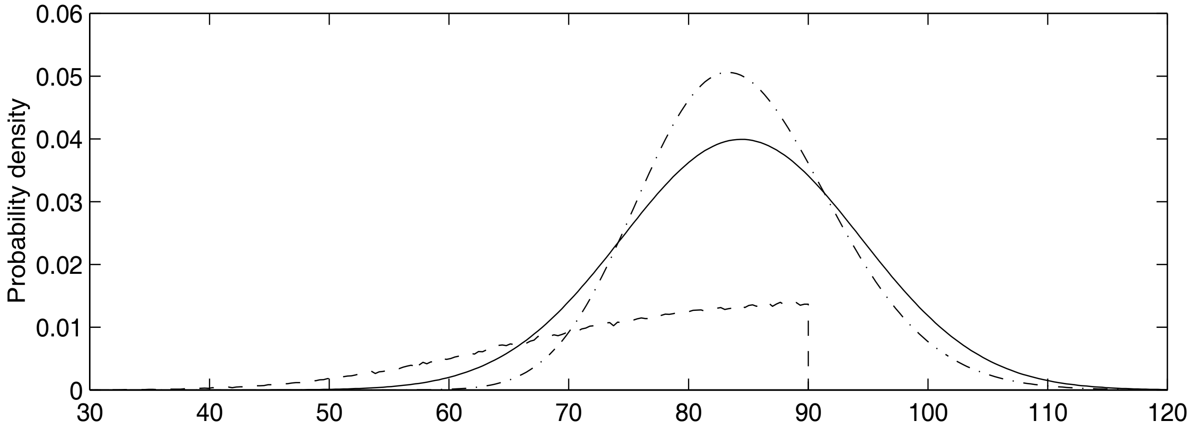
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| | $\sigma_{L_{i0}}$ | $\sigma_{L_0}$ | $L_0$ | $B_T$ | $P(\cdot)$ | $P(\cdot)/B_0(\%)$ | $Q(\cdot)$ | $Q(\cdot)/B_0(\%)$ | $\phi$ |
| Base | 0.0942 | 0.0669 | 80.30 | 72.27 | 0.0171 | 0.0249 | 0.3189 | 0.4638 | 5.3687 |
| $r = 0.09$ | 0.0813 | 0.0578 | 78.06 | 70.25 | 0.0004 | 0.0005 | 0.1303 | 0.1949 | 0.2565 |
| $r = 0.07$ | 0.0876 | 0.0623 | 79.19 | 71.27 | 0.0033 | 0.0048 | 0.2066 | 0.3049 | 1.5743 |
| $r = 0.03$ | 0.1009 | 0.0717 | 81.39 | 73.25 | 0.0623 | 0.0895 | 0.4775 | 0.6853 | 13.0600 |
| $r = 0.01$ | 0.1079 | 0.0767 | 82.46 | 74.22 | 0.1728 | 0.2447 | 0.6987 | 0.9897 | 24.7247 |
| $T = 2.0$ | 0.0989 | 0.0703 | 73.76 | 66.38 | 0.0469 | 0.0743 | 0.4982 | 0.7889 | 9.4182 |
| $T = 1.5$ | 0.0976 | 0.0694 | 76.82 | 69.14 | 0.0362 | 0.0550 | 0.4350 | 0.6614 | 8.3157 |
| $T = 0.5$ | 0.0836 | 0.0594 | 84.51 | 76.06 | 0.0009 | 0.0013 | 0.1296 | 0.1790 | 0.7263 |
| $T = 0.25$ | 0.0670 | 0.0476 | 87.14 | 78.43 | 0.0000 | 0.0000 | 0.0277 | 0.0372 | 0.0000 |
| $F_{iT} = 8.5$ | 0.0760 | 0.0540 | 77.08 | 69.37 | 0.0023 | 0.0035 | 0.1981 | 0.3001 | 1.1663 |
| $F_{iT} = 8.0$ | 0.0589 | 0.0419 | 73.54 | 66.18 | 0.0001 | 0.0001 | 0.1106 | 0.1757 | 0.0569 |
| $F_{iT} = 7.5$ | 0.0435 | 0.0309 | 69.70 | 62.73 | 0.0000 | 0.0000 | 0.0541 | 0.0906 | 0.0000 |
| $F_{iT} = 7.0$ | 0.0302 | 0.0215 | 65.60 | 59.04 | 0.0000 | 0.0000 | 0.0226 | 0.0403 | 0.0000 |
| $\sigma_i = 0.45$ | 0.1698 | 0.1207 | 75.33 | 67.80 | 0.3928 | 0.6090 | 1.1572 | 1.7943 | 33.9408 |
| $\sigma_i = 0.40$ | 0.1445 | 0.1027 | 77.02 | 69.31 | 0.2077 | 0.3151 | 0.8432 | 1.2789 | 24.2590 |
| $\sigma_i = 0.35$ | 0.1192 | 0.0847 | 78.68 | 70.81 | 0.0808 | 0.1200 | 0.5580 | 0.8284 | 14.4858 |
| $\sigma_i = 0.20$ | 0.0457 | 0.0325 | 83.30 | 74.97 | 0.0000 | 0.0000 | 0.0376 | 0.0527 | 0.0000 |
| $\rho_{ij} = 0.8$ | 0.0942 | 0.0843 | 80.30 | 72.27 | 0.0804 | 0.1169 | 0.8303 | 1.2077 | 9.6796 |
| $\rho_{ij} = 0.6$ | 0.0942 | 0.0732 | 80.30 | 72.27 | 0.0330 | 0.0480 | 0.4705 | 0.6844 | 7.0134 |
| $\rho_{ij} = 0.3$ | 0.0942 | 0.0522 | 80.30 | 72.27 | 0.0016 | 0.0023 | 0.0874 | 0.1271 | 1.8096 |
| $\rho_{ij} = 0.0$ | 0.0942 | 0.0094 | 80.30 | 72.27 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | — |
| $B_T/L_0 = 0.85$ | 0.0942 | 0.0669 | 80.30 | 68.26 | 0.0010 | 0.0015 | 0.1464 | 0.2255 | 0.6652 |
| $B_T/L_0 = 0.80$ | 0.0942 | 0.0669 | 80.30 | 64.24 | 0.0000 | 0.0000 | 0.0611 | 0.1000 | 0.0000 |
| $B_T/L_0 = 0.75$ | 0.0942 | 0.0669 | 80.30 | 60.23 | 0.0000 | 0.0000 | 0.0228 | 0.0397 | 0.0000 |
| $B_T/L_0 = 0.70$ | 0.0942 | 0.0669 | 80.30 | 56.21 | 0.0000 | 0.0000 | 0.0074 | 0.0138 | 0.0000 |

Table 3: **Capital Adequacy Requirement Under the Traditional Approach and the Dis-Aggregated one that Considers the Individual Loans' Default Risks**. Column 1 reports the alternative parameter values considered for the specific simulation. Panel A, B and C consider the cases when the insurance premium is 0.25%, 0.50% and 0.75% of the deposits, respectively. Specifically, column 2 reports the insurance premium under the traditional approach. Column 3 reports the equity percent of the total asset such that the insurance premium is 0.25% of the deposits. Columns 4 and 5 report the corresponding numbers under the dis-aggregated approach, respectively. Columns under Panel B and C have similar interpretations. The number of borrowing firms, $n$, is fixed at 10.

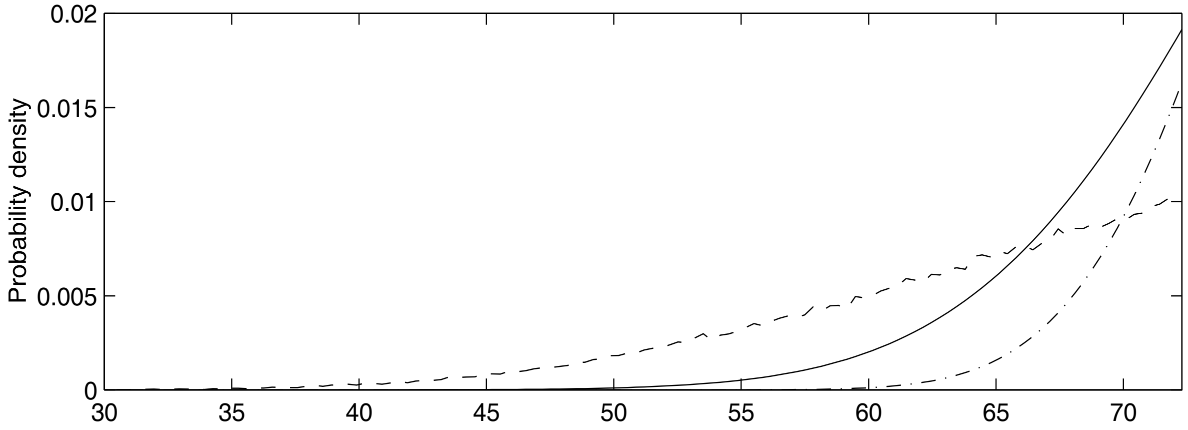
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Panel A: Premium is 0.25% | | | | Panel B: Premium is 0.50% | | | | Panel C: Premium is 0.75% | | | |
| | $P(\cdot)$ | $\frac{E_P}{L_0}$ (%) | $Q(\cdot)$ | $\frac{E_Q}{L_0}$ (%) | $P(\cdot)$ | $\frac{E_P}{L_0}$ (%) | $Q(\cdot)$ | $\frac{E_Q}{L_0}$ (%) | $P(\cdot)$ | $\frac{E_P}{L_0}$ (%) | $Q(\cdot)$ | $\frac{E_Q}{L_0}$ (%) |
| Base | 0.182 | 9.52 | 0.160 | 20.26 | 0.371 | 7.39 | 0.339 | 15.38 | 0.567 | 5.96 | 0.528 | 12.26 |
| $r = 0.09$ | 0.179 | 7.89 | 0.159 | 18.21 | 0.366 | 5.96 | 0.338 | 13.33 | 0.560 | 4.66 | 0.523 | 10.30 |
| $r = 0.07$ | 0.181 | 8.69 | 0.161 | 19.19 | 0.369 | 6.66 | 0.341 | 14.31 | 0.562 | 5.30 | 0.526 | 11.28 |
| $r = 0.03$ | 0.183 | 10.39 | 0.161 | 21.24 | 0.374 | 8.15 | 0.340 | 16.36 | 0.569 | 6.66 | 0.529 | 13.23 |
| $r = 0.01$ | 0.182 | 11.30 | 0.160 | 22.22 | 0.375 | 8.94 | 0.340 | 17.33 | 0.573 | 7.37 | 0.528 | 14.21 |
| $T = 2.0$ | 0.156 | 15.46 | 0.127 | 31.40 | 0.321 | 12.66 | 0.276 | 25.44 | 0.494 | 10.79 | 0.436 | 21.53 |
| $T = 1.5$ | 0.167 | 12.81 | 0.140 | 26.51 | 0.345 | 10.27 | 0.303 | 20.95 | 0.527 | 8.61 | 0.477 | 17.33 |
| $T = 0.5$ | 0.200 | 5.15 | 0.187 | 11.67 | 0.407 | 3.62 | 0.387 | 7.96 | 0.617 | 2.59 | 0.597 | 5.62 |
| $T = 0.25$ | 0.213 | 2.23 | 0.206 | 5.52 | 0.429 | 1.22 | 0.426 | 2.88 | 0.649 | 0.53 | 0.636 | 1.42 |
| $F_{iT} = 8.5$ | 0.179 | 7.22 | 0.160 | 17.24 | 0.365 | 5.38 | 0.336 | 12.45 | 0.554 | 4.16 | 0.523 | 9.42 |
| $F_{iT} = 8.0$ | 0.174 | 5.13 | 0.158 | 13.92 | 0.354 | 3.60 | 0.335 | 9.23 | 0.538 | 2.58 | 0.514 | 6.49 |
| $F_{iT} = 7.5$ | 0.169 | 3.31 | 0.157 | 10.30 | 0.342 | 2.09 | 0.325 | 6.10 | 0.515 | 1.28 | 0.500 | 3.66 |
| $F_{iT} = 7.0$ | 0.161 | 1.88 | 0.154 | 6.69 | 0.327 | 0.94 | 0.315 | 3.18 | 0.493 | 0.30 | 0.485 | 1.22 |
| $\sigma_i = 0.45$ | 0.151 | 19.39 | 0.121 | 36.08 | 0.316 | 16.19 | 0.263 | 30.13 | 0.486 | 14.10 | 0.419 | 26.12 |
| $\sigma_i = 0.40$ | 0.161 | 16.08 | 0.134 | 31.01 | 0.336 | 13.18 | 0.287 | 25.34 | 0.512 | 11.31 | 0.455 | 21.53 |
| $\sigma_i = 0.35$ | 0.171 | 12.77 | 0.147 | 25.73 | 0.354 | 10.23 | 0.314 | 20.36 | 0.540 | 8.57 | 0.488 | 16.94 |
| $\sigma_i = 0.20$ | 0.202 | 3.56 | 0.188 | 8.93 | 0.407 | 2.30 | 0.391 | 5.52 | 0.615 | 1.45 | 0.599 | 3.47 |
| $\rho_{ij} = 0.8$ | 0.176 | 12.22 | 0.141 | 29.25 | 0.363 | 9.75 | 0.308 | 23.58 | 0.553 | 8.13 | 0.483 | 19.87 |
| $\rho_{ij} = 0.6$ | 0.179 | 10.48 | 0.154 | 23.39 | 0.368 | 8.22 | 0.330 | 18.12 | 0.563 | 6.71 | 0.516 | 14.70 |
| $\rho_{ij} = 0.3$ | 0.186 | 7.37 | 0.174 | 13.82 | 0.379 | 5.52 | 0.363 | 10.01 | 0.577 | 4.28 | 0.557 | 7.67 |
| $\rho_{ij} = 0.0$ | 0.195 | 2.94 | 0.188 | 5.03 | 0.393 | 1.80 | 0.387 | 3.27 | 0.598 | 1.01 | 0.580 | 2.20 |

Table 4: **Capital Adequacy Requirement for Different Number of Loans and Different Levels of Correlation Across Borrowers**. Column 1 reports the alternative parameter values. Columns 2-14 report the capital requirement for different correlation coefficients, $\rho_{ij}$, *i.e.,* the equity-total asset ratio, $E_Q/L_0$ (%), that the bank should maintain to ensure that the deposit insurance premium $Q(\cdot)/B_0$ (%), which is set at 1.7117, is actuarially fair for different levels of $\rho_{ij}$.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_{ij}$ | 0.60 | 0.55 | 0.50 | 0.45 | 0.40 | 0.35 | **0.30** | 0.25 | 0.20 | 0.15 | 0.10 | 0.05 | 0.00 |
| | Panel A: Equity Cushion, $E_Q/L_0$ (%), when $n = 10$ | | | | | | | | | | | | |
| base | 19.34 | 18.04 | 16.77 | 15.40 | 14.06 | 12.71 | **11.43** | 10.08 | 8.81 | 7.55 | 6.26 | 5.00 | 3.76 |
| $r = 0.09$ | 19.26 | 17.98 | 16.71 | 15.36 | 14.05 | 12.72 | **11.48** | 10.15 | 8.92 | 7.69 | 6.43 | 5.21 | 4.01 |
| $r = 0.07$ | 19.30 | 18.01 | 16.74 | 15.38 | 14.05 | 12.72 | **11.45** | 10.11 | 8.87 | 7.62 | 6.35 | 5.10 | 3.89 |
| $r = 0.03$ | 19.40 | 18.09 | 16.80 | 15.42 | 14.06 | 12.71 | **11.41** | 10.04 | 8.76 | 7.48 | 6.16 | 4.88 | 3.62 |
| $r = 0.01$ | 19.45 | 18.13 | 16.82 | 15.44 | 14.08 | 12.70 | **11.39** | 10.00 | 8.71 | 7.41 | 6.07 | 4.76 | 3.49 |
| $T = 2.0$ | 19.73 | 18.25 | 16.83 | 15.35 | 13.83 | 12.32 | **10.87** | 9.37 | 7.96 | 6.52 | 5.06 | 3.62 | 2.20 |
| $T = 1.5$ | 19.62 | 18.21 | 16.84 | 15.40 | 13.95 | 12.51 | **11.13** | 9.69 | 8.34 | 6.98 | 5.59 | 4.23 | 2.89 |
| $T = 0.5$ | 18.82 | 17.67 | 16.53 | 15.34 | 14.13 | 12.93 | **11.80** | 10.60 | 9.44 | 8.32 | 7.16 | 6.03 | 4.93 |
| $T = 0.25$ | 18.40 | 17.37 | 16.35 | 15.30 | 14.20 | 13.12 | **12.06** | 10.96 | 9.87 | 8.88 | 7.78 | 6.74 | 5.76 |
| $F_{iT} = 8.5$ | 19.22 | 17.96 | 16.70 | 15.35 | 14.05 | 12.73 | **11.49** | 10.19 | 8.96 | 7.75 | 6.51 | 5.30 | 4.13 |
| $F_{iT} = 8.0$ | 19.12 | 17.88 | 16.65 | 15.33 | 14.05 | 12.75 | **11.56** | 10.29 | 9.10 | 7.95 | 6.75 | 5.60 | 4.48 |
| $F_{iT} = 7.5$ | 19.04 | 17.81 | 16.60 | 15.31 | 14.05 | 12.79 | **11.64** | 10.41 | 9.24 | 8.13 | 6.99 | 5.89 | 4.83 |
| $F_{iT} = 7.0$ | 18.97 | 17.77 | 16.56 | 15.31 | 14.07 | 12.82 | **11.71** | 10.52 | 9.39 | 8.33 | 7.22 | 6.18 | 5.17 |
| | Panel B: Equity Cushion, $E_Q/L_0$ (%), when $n = 100$ | | | | | | | | | | | | |
| base | 22.27 | 20.57 | 18.85 | 17.07 | 15.26 | 13.43 | **11.59** | 9.72 | 7.82 | 5.91 | 3.93 | 1.87 | 0.00 |
| $r = 0.09$ | 22.23 | 20.54 | 18.83 | 17.07 | 15.26 | 13.45 | **11.63** | 9.78 | 7.90 | 6.01 | 4.07 | 2.04 | 0.00 |
| $r = 0.07$ | 22.25 | 20.55 | 18.84 | 17.06 | 15.25 | 13.44 | **11.61** | 9.75 | 7.86 | 5.96 | 4.00 | 1.95 | 0.00 |
| $r = 0.03$ | 22.29 | 20.59 | 18.87 | 17.08 | 15.25 | 13.43 | **11.57** | 9.69 | 7.78 | 5.85 | 3.87 | 1.79 | 0.00 |
| $r = 0.01$ | 22.32 | 20.62 | 18.88 | 17.09 | 15.26 | 13.42 | **11.55** | 9.66 | 7.75 | 5.80 | 3.80 | 1.69 | 0.00 |
| $T = 2.0$ | 22.95 | 21.06 | 19.14 | 17.17 | 15.17 | 13.14 | **11.11** | 9.05 | 6.96 | 4.86 | 2.71 | 0.47 | 0.00 |
| $T = 1.5$ | 22.70 | 20.88 | 19.04 | 17.15 | 15.23 | 13.29 | **11.31** | 9.36 | 7.35 | 5.34 | 3.26 | 1.10 | 0.00 |
| $T = 0.5$ | 21.54 | 20.02 | 18.48 | 16.88 | 15.26 | 13.59 | **11.91** | 10.19 | 8.44 | 6.66 | 4.81 | 2.86 | 0.67 |
| $T = 0.25$ | 20.89 | 19.54 | 18.16 | 16.71 | 15.23 | 13.71 | **12.12** | 10.52 | 8.84 | 7.12 | 5.34 | 3.47 | 1.33 |
| $F_{iT} = 8.5$ | 22.22 | 20.53 | 18.82 | 17.06 | 15.26 | 13.46 | **11.64** | 9.80 | 7.94 | 6.05 | 4.12 | 2.11 | 0.00 |
| $F_{iT} = 8.0$ | 22.23 | 20.51 | 18.82 | 17.06 | 15.28 | 13.50 | **11.70** | 9.89 | 8.04 | 6.19 | 4.30 | 2.34 | 0.19 |
| $F_{iT} = 7.5$ | 22.25 | 20.53 | 18.85 | 17.08 | 15.32 | 13.54 | **11.77** | 9.97 | 8.15 | 6.33 | 4.47 | 2.55 | 0.45 |
| $F_{iT} = 7.0$ | 22.29 | 20.57 | 18.88 | 17.13 | 15.38 | 13.59 | **11.83** | 10.05 | 8.25 | 6.46 | 4.64 | 2.75 | 0.71 |

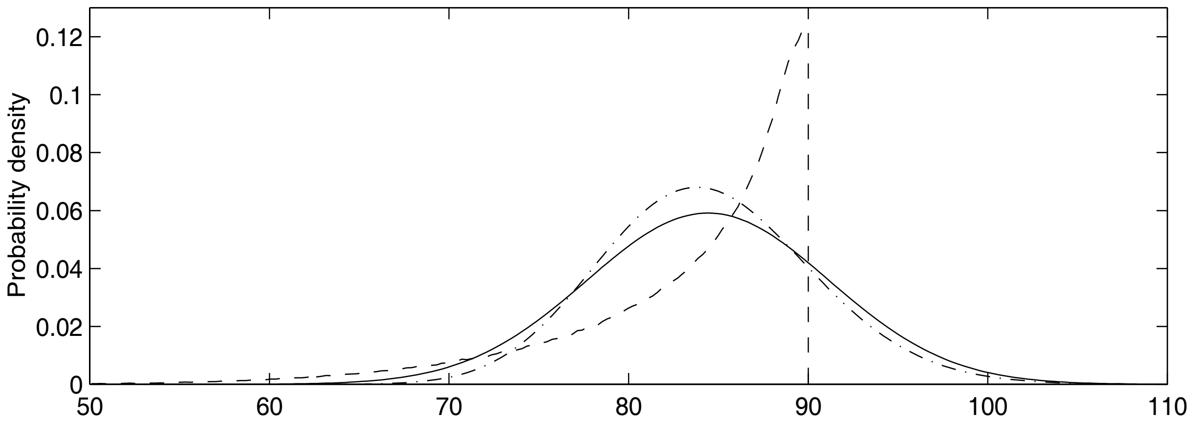


A: Complete density distribution of loan portfolio value (n=1)

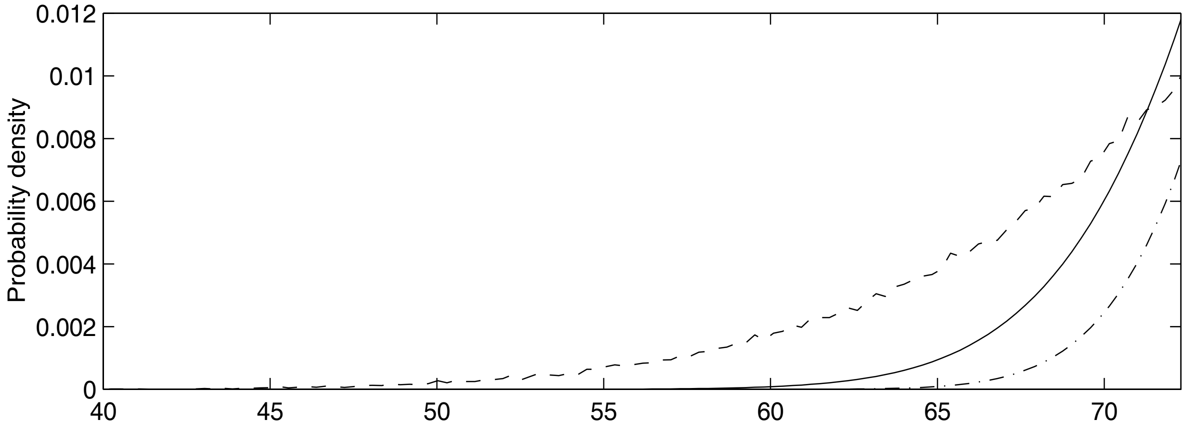


B: Tail density distribution for loan portfolio value < face value of total deposits (72.27), (n=1)

Figure 1: **Distribution of terminal loan portfolio value of 1 loan.** The solid and dashed lines represent the normal and lognormal distributions, respectively. The dotted line represents the true distribution generated by 1 million Monte Carlo simulations. The parameter values correspond to the base case values in Table 1 except $n = 1$. The initial asset value and the face value of the loan are scaled so that the loan portfolio value is kept the same at 80.30. The point mass probability at 90 is 0.6452.

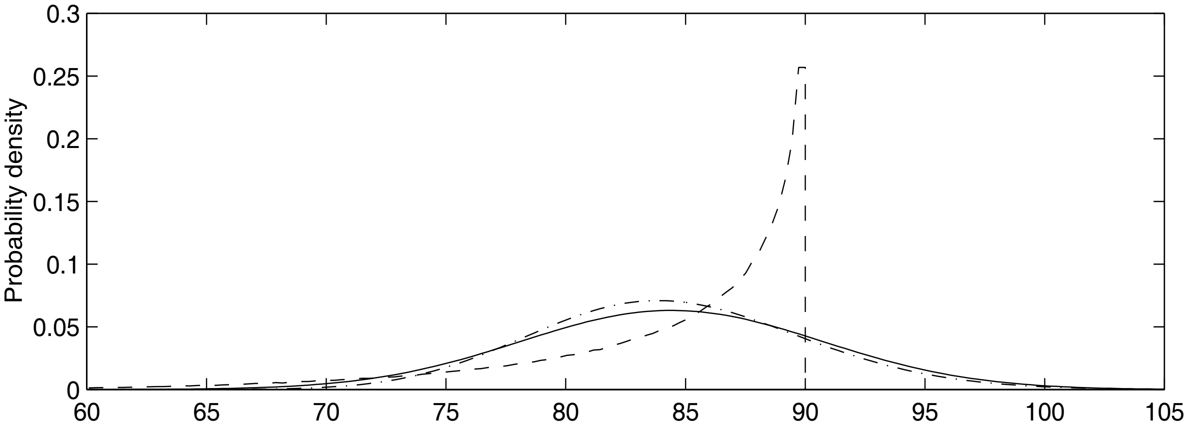


A: Complete density distribution of loan portfolio value (n=10)

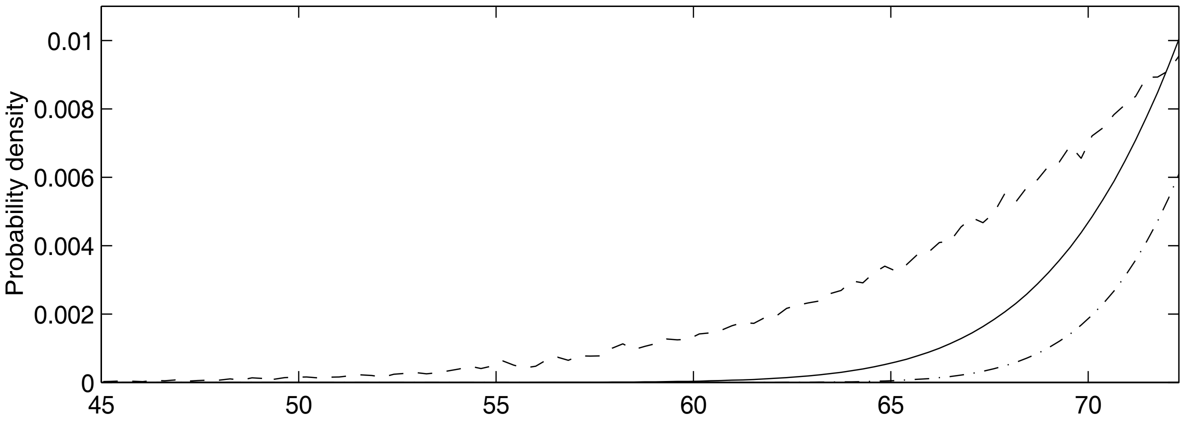


B: Tail density distribution for loan portfolio value < face value of total deposits (72.27), (n=10)

Figure 2: **Distribution of terminal loan portfolio value of 10 correlated loans.** The solid and dashed lines represent the normal and lognormal distributions, respectively. The dotted line represents the true distribution generated by 1 million Monte Carlo simulations. The parameter values correspond to the base case values in Table 1. The point mass probability at 90 is 0.2076.

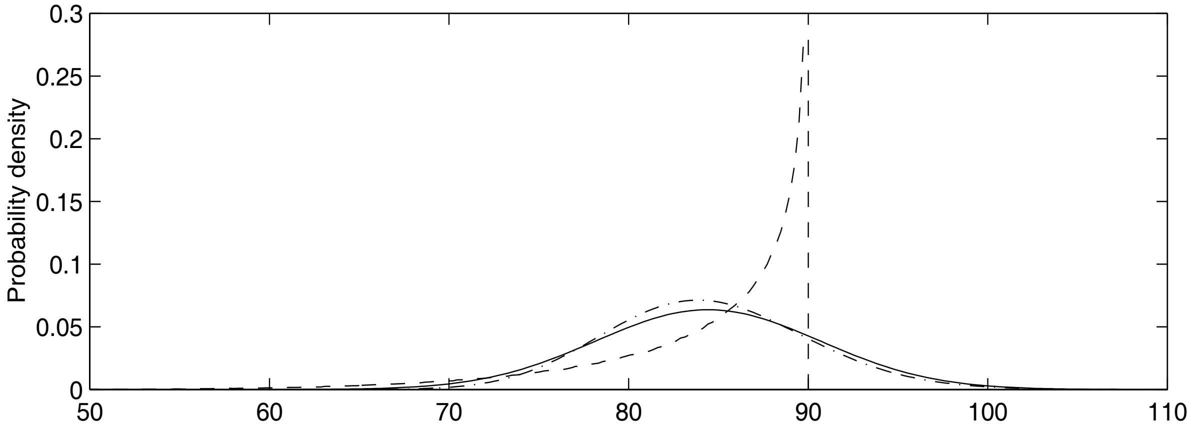


A: Complete density distribution of loan portfolio value (n=100)

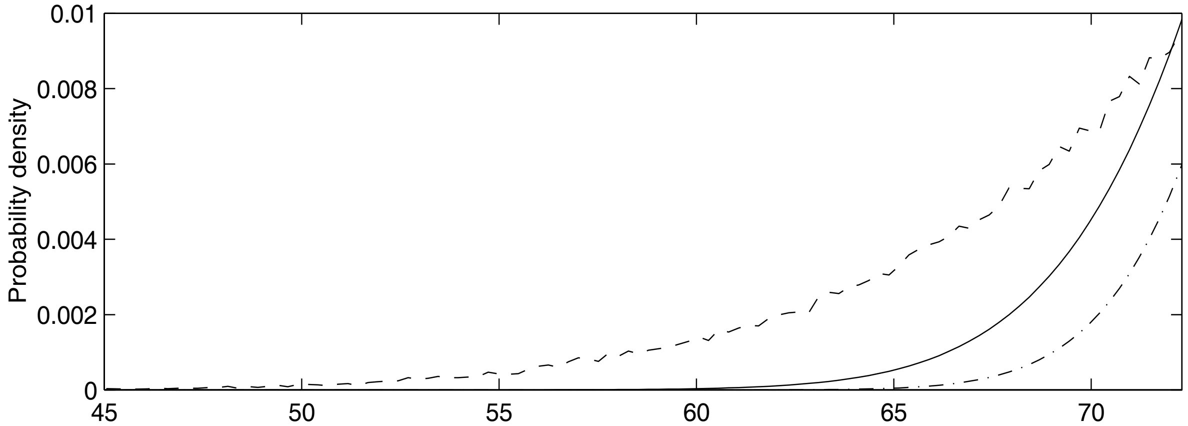


B: Tail density distribution for loan portfolio value < face value of total deposits (72.27), (n=100)

Figure 3: **Distribution of terminal loan portfolio value of 100 correlated loans.** The solid and dashed lines represent the normal and lognormal distributions, respectively. The dotted line represents the true distribution generated by 1 million Monte Carlo simulations. The parameter values correspond to the base case values in Table 1 except $n = 100$. The initial asset values and the face values of the loans are scaled so that the loan portfolio value is kept the same at 80.30. The point mass probability at 90 is 0.08.

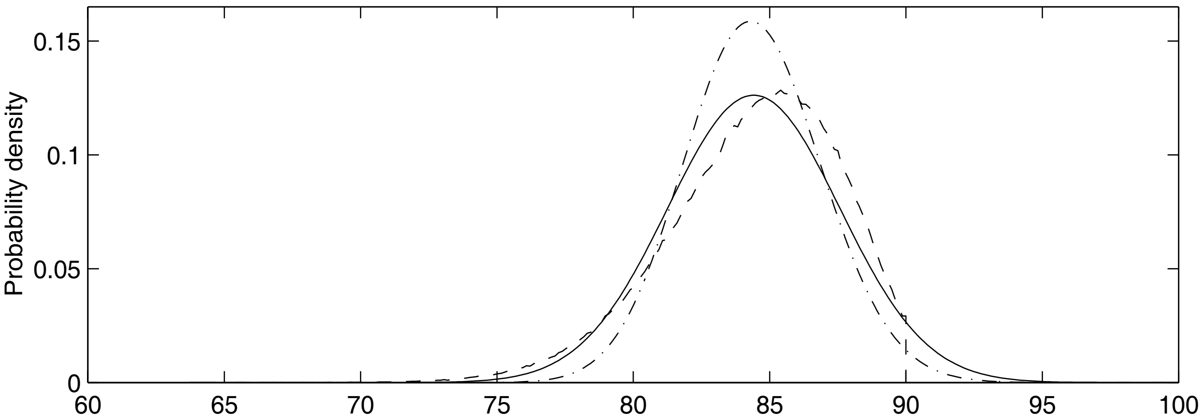


A: Complete density distribution of loan portfolio value (n=1000)

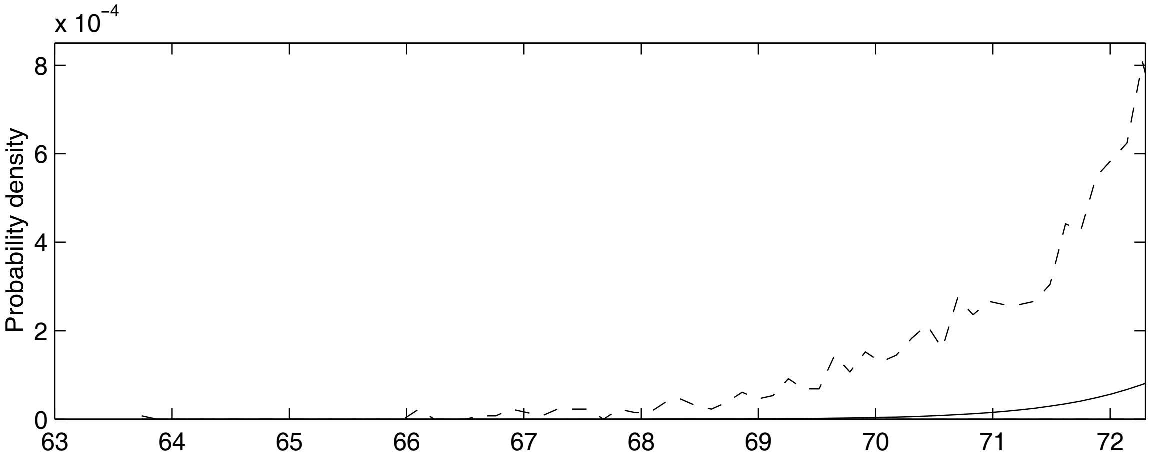


B: Tail density distribution for loan portfolio value < face value of total deposits (72.27), (n=1000)

Figure 4: **Distribution of terminal loan portfolio value of 1,000 correlated loans.** The solid and dashed lines represent the normal and lognormal distributions, respectively. The dotted line represents the true distribution generated by 1 million Monte Carlo simulations. The parameter values correspond to the base case values in Table 1 except $n = 1,000$. The initial asset values and the face values of the loans are scaled so that the loan portfolio value is kept the same at 80.30. The point mass probability at 90 is 0.0629.

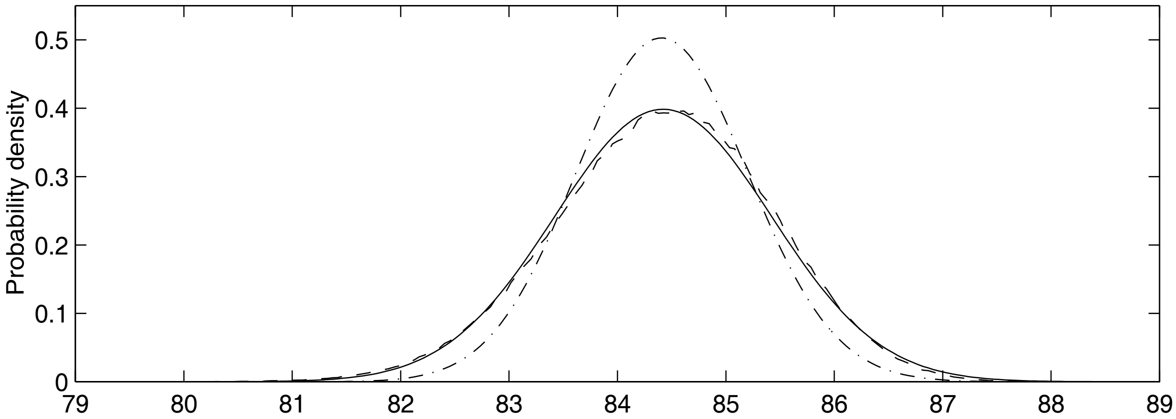


A: Complete density distribution of loan portfolio value (n=10)

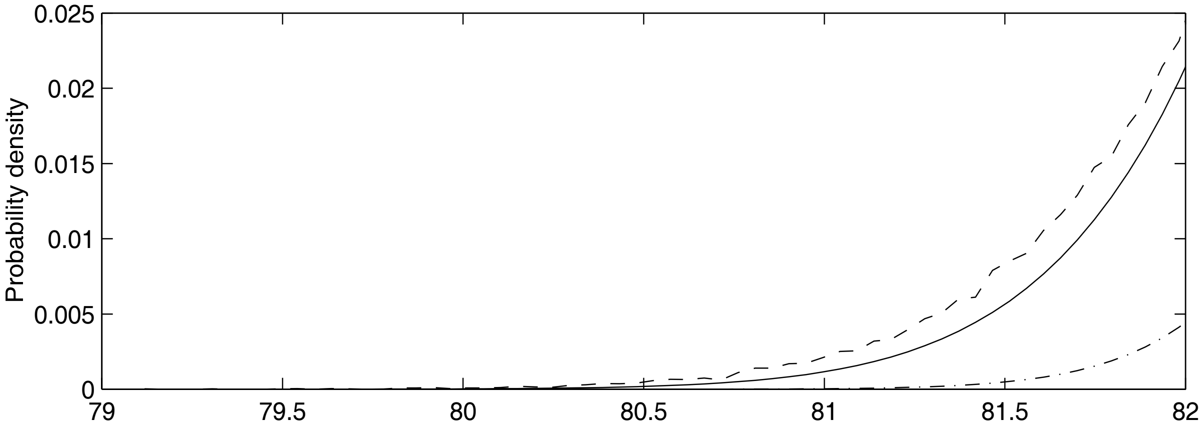


B: Tail density distribution for loan portfolio value < face value of total deposits (72.27), (n=10)

Figure 5: **Distribution of terminal loan portfolio value of 10 independent loans.** The solid and dashed lines represent the normal and lognormal distributions, respectively. The dotted line represents the true distribution generated by 1 million Monte Carlo simulations. The parameter values correspond to the base case values in Table 1 except the correlation coefficient $\rho_{ij} = 0$. The point mass probability at 90 is $0.6452^{10} = 0.0125$, where 0.6452 is the mass probability at 90 for one loan.



A: Complete density distribution of loan portfolio value (n=100)



B: Tail density distribution for loan portfolio value between 79 and 82 (n=100)

Figure 6: **Distribution of terminal loan portfolio value of 100 independent loans.** The solid and dashed lines represent the normal and lognormal distributions, respectively. The dotted line represents the true distribution generated by 1 million Monte Carlo simulations. The parameter values correspond to the base case values in Table 1 except $n = 100$ and the correlation coefficient $\rho_{ij} = 0$. The initial asset values and the face values of the loans are scaled so that the loan portfolio value is kept the same at 80.30. The point mass probability at 90 is $0.6452^{100} \approx 0$, where 0.6452 is the mass probability at 90 for one loan.