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Optimal workpiece setups for 4-axis numerical control machining based on machinability

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Abstract

Orienting of a workpiece so as to maximize the number of surfaces machined on a 4-axis Numerical Control (NC) machine is formulated as a spherical computational geometry problem. The orientations along which a sampling point on the part surface is visible to a 3-axis NC machine are represented as a spherical polygon. The range of rotation of the fourth axis of a 4-axis NC machine is assumed to be 180°s and is represented as a semi-great circle. The maximization of the number of surfaces machined is formulated as finding a semi-great circle that intersects the maximal number of spherical polygons each of which corresponds to a sample point on the surface of the workpiece. This paper provides an $O((E+I_{wb})^2N)$ time algorithm for this optimization problem, where N is the number of spherical polygons, E is the total number of polygonal edges, and I_{wb} is the number of pairs of polygonal edges that are antipodal to each other. The solution to this maximization problem helps establishing a heuristic to a workpiece setup optimization problem, in which the number of setups for a workpiece to be machined by a 4-axis NC machine is to be minimized. In addition to the development of the algorithm, useful properties of geometric duality are also established. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: NC machining; Machinability; Setup orientation; Geometric algorithms

1. Introduction

One of the main tasks in planning machining processes for a mechanical part is determination of the setup mounting of the workpiece with a certain orientation and fixturing on the work table. Workpiece setup is time-consuming as mounting and dismounting of the workpiece are usually done manually, and is expensive if specially designed fixtures and clamps may need to be used. Moreover, potential discrepancies in the settings of the machine coordinate system are often a result of multiple setups. It is therefore desirable to have a minimal number of setups. In this paper, an algorithm is developed to aid the determination of the minimal number of workpiece setups for 4-axis NC machining.

Since a part is often represented by a group of surfaces in a CAD system, the manufacture of which can be considered as the machining of its surfaces. For a surface to be machinable, every point in the surface has to be accessible by the tool. It is established in Refs. [1,2] that for a tool to be able to access an entire surface, the axis of the tool cannot deviate from the normal at any point in the surface

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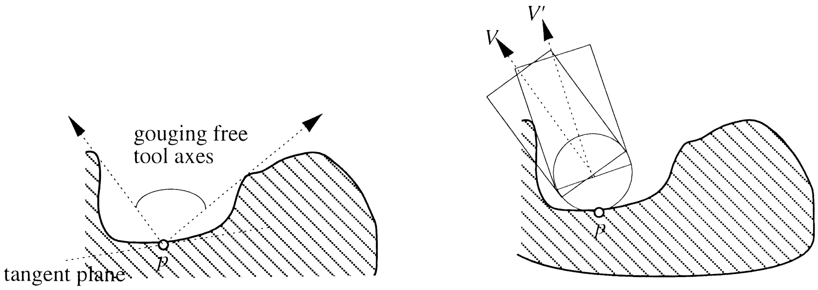

(a) (b)

Fig. 1. Accessibility of a point to a tool.

by more than 90°s. Depending on the shape of the tool [1,2] the allowable deviation from the normal may even be smaller than 90°. With a *ball-end mill*, the tool can deviate from the surface normal by up to 90°; with a *fillet-end mill*, the tool can only deviate from the surface normal by an angle of less than 90°; with a *flat-end mill*, the allowable deviation is the minimum. [1] In the subsequent discussion, the ball-end mill is considered.

The accessible orientations of the tool axis are further reduced. When the size of the tool is negligible, the accessible orientations for every point in the surface are obtained by computing the set of directions in which the point is visible from infinity, as illustrated in Fig. 1a. Otherwise, if the geometry of a tool is not negligible, the accessible orientations can be computed by dilating the surface of the part while eroding the tool to a line segment [3], as shown in Fig. 1b. This reduces the latter case to the formal case where the geometry of the tool is negligible.

The resulting accessible orientations of a tool such that it can access a surface point after considering both gouging and the geometry of the tool form a *visibility cone*, which is unbounded since an accessible orientation implies that a tool can approach the point along this orientation from infinity. By translating the apex of such a cone to the origin and intersecting the cone with a unit sphere, a spherical map is obtained, which is called the *visibility map* or a VMap of the point [2]. This VMap represents all the orientations along which a given tool can access and machine the point on the surface.

In practice, due to the complexity of sculptured surfaces, the machinability of a part is often verified by checking a finite set of sample points on the part surface. Algorithms for generating grid points as sample points can be found in Ref. [4]. In contrast to the sampling approach, recent research has reported on the clustering of the lower-order surfaces [5]. The common intersection of the VMaps for points on a single surface is called the VMap of the surface, which represents the orientations for the tool to machine the entire surface. However, such kind of computation can easily become overwhelming for higher-order surfaces when gouging is considered. This paper thus focuses on the sampling approach with the following exception: For geometric primitives, such as cylindrical and planar features, the accessible orientations are computed as a whole rather than by sample points and associated with weights proportional to their individual surface areas.

Implicit to the previous discussion of visibility is a 3-axis milling machine, that is, a tool for a 3-axis machine ‘sees’ up to a hemisphere. A part is machinable by a 3-axis machine if and only if the common intersection of the VMaps of all the sample points is non-empty. While a 3-axis machine can only provide a single orientation for the tool axis, a 4-axis machine supports an infinite number of tool orientations through the rotation of the work table about the fourth axis, which is usually parallel to one of the three principal axes, e.g., the *x*-axis on the 4-axis NC machine as shown in Fig. 2. A mechanical part that cannot be machined in one setup with a 3-axis NC machine might be able to be machined in one

[1] When performing pocket machining, the bottom as well as the walls whose normals are perpendicular to the cutter-axis are machined by flat-end mills. Additionally, curved surfaces can also be machined by flat-end mills with an inclination angle.

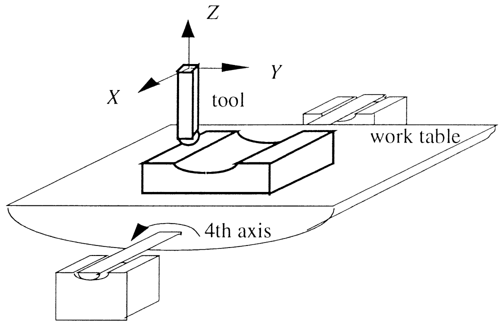

Fig. 2. A 4-axis milling machine.

setup with a 4-axis machine. For example, if the part in Fig. 3 is mounted so that it can be rotated by rotation R_a, then every surface of the part can be rotated to a position such that the entire surface is accessible to a tool coming down from the z direction. If the setup orientation is not properly chosen, however, the reduction of the number of setups may not be achieved even with a 4-axis machine. An example is rotation R_b shown in Fig. 3.

Since a workpiece is fixed relative to the fourth axis, the rotation about the fourth axis gives rise to setup orientations orthogonal to the rotational axis. Consequently, the set of tool axes for a 4-axis NC machine can be represented by a great circle on the unit sphere, as illustrated in Fig. 4. Since the attainable rotation of the work table is usually an angle θ_r of less than 360°s, the set of tool orientations is represented as an arc of radial angle θ_r on the unit sphere, called an *eigen arc*.

By utilizing the notions of eigen arcs and VMaps, the problem of minimizing the number of setups for a workpiece (to be machined by a 4-axis machine) can be formalized as follows.

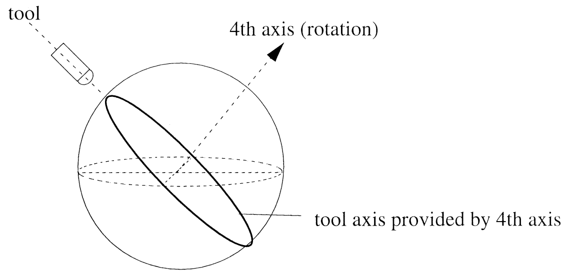

Fig. 4. A representation of a tool axis provided by the fourth axis.

Problem. Minimal number of workpiece setups on a 4-axis machine. Given a set of VMaps, each of which corresponds to the accessible orientations for the tool to a sample point on a 3-axis machine, find a set of eigen arcs with a minimal cardinality such that every VMap intersects at least one of the eigen arcs in the set.

This optimization problem is NP-complete because the case where θ_r equals zero is known to be NP-complete [6]. Heuristic solutions are therefore useful. This paper presents a greedy approach that seeks an eigen arc, called a *maximal cutting arc*, that intersects a set of VMaps with the maximal weight. Using the greedy approach at each iteration, a maximal cutting arc is computed and the VMaps intersected by the maximal cutting arc are eliminated from the given set of VMaps. This process is repeated until the set becomes empty. This greedy approach seeks to set up a workpiece in such a way that a maximal number of surfaces is machined by a 4-axis machine.

The rest of the paper is organized as follows. Section 2 describes the techniques used in the greedy

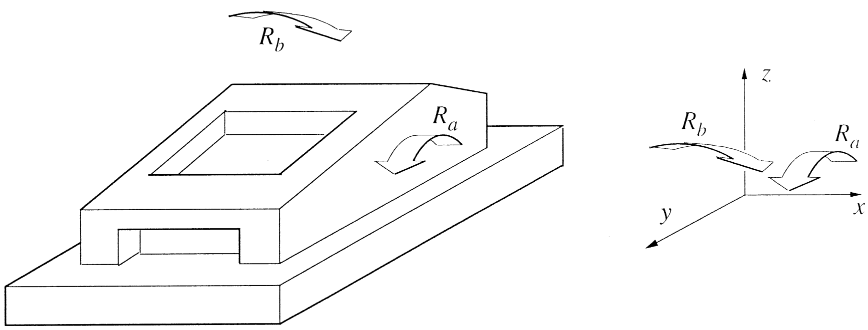

Fig. 3. A part that can be machined in one setup with a 4-axis NC machine.

approach and briefly reviews the geometric duality in the plane. Section 3 utilizes this duality to find a maximal cutting arc for a special case where all the VMaps are line segments. In Section 4, the result of Section 3 is generalized for VMaps represented by simple polygons. The algorithm developed in this paper for finding a maximal cutting arc runs in $O((E+I_{wb})^2 N)$ time, where N is the number of polygons that have a total number of E vertices, and I_{wb} is the number of intersections between the edges of polygons of upper or lower hemispheres.

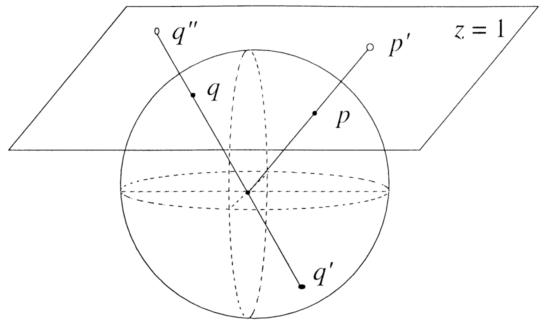

Fig. 5. Central projection.

2. Solution approach

The maximal intersection problem itself exhibits many difficulties, among which the most troublesome one is the search domain being continuous, that is, there may be an infinite number of eigen arcs that intersect the same maximal number of VMaps. Tang and Woo [7] investigated the case where the angle θ_r is 360° and the VMaps are represented as spherical polygons, and reported an $O(EN^2+N^3 \log N)$ time and $O(E+N^2)$ space algorithm, where N is the number of spherical polygons and E is the total number of edges. These two upper-bounds are recently further improved by Gupta et al. [8] to $O(E^2)$ time and $O(NE)$ space respectively. However, as indicated above, by setting θ_r to 360°, the tool may interfere with the work table.

In this paper, the angle θ_r is assumed to be equal to 180° since for some machine configurations the axis of a tool cannot deviate from the upward pointing normal of the (presumably flat) work table by more than 90°. The maximal cutting problem under this assumption is formalized as follows.

Problem. Maximal cutting with a semi-great circle. Given a set of spherical polygons, find a semi-great circle that intersects the maximal number of them.

An algorithm is developed for solving this optimization problem. Prior to the development of the algorithm, two transformations are performed on the input: one transforms the spherical problem to a planar problem; the other establishes this planar problem in a dual domain.

To avoid dealing with the non-linearity of arcs, the problem is transformed from the spherical domain to the planar domain by using the augmented central projection [1]. Under the central projection, a point p on the unit sphere has a unique image point p' in the $z=1$ plane. The image point p' can be obtained by intersecting the plane $z=1$ with the ray emanating from the center of the unit sphere and passing through p, as shown in Fig. 5. However, a pair of antipodal [2] points will be mapped to the same point in the plane, as manifested by q, q', and q'' in the figure. Noting this two-to-one nature of the mapping, image points are colored as white or black to distinguish whether they are images of points on the upper- or lower-hemisphere.

With such a classification, the image of a semi-great circle, which is a straight line, would correspond to two opposite half-lines of different colors: the white one being the image of the portion from the upper-hemisphere, and the black one the portion from the lower-hemisphere. The intersection of the two half-lines is the image of the two (antipodal) endpoints of the semi-great circle, as shown in Fig. 6. Additionally, the image of a spherical polygon that has no intersection with the equator $(z=0)$, is a bounded polygon of a single color in the plane; if one of the edges of the spherical polygon lies on the equator, its image is an unbounded polygon of a single color; if the spherical polygon crosses the equator, its image is two disjoint [3] and unbounded

[2] Two points on the sphere are antipodal to each other if they are the end points of a diameter.

[3] Since the spherical polygons in this paper are all hemispherical, the two unbounded polygons resulting from the projection of the same spherical polygon are disjoint.

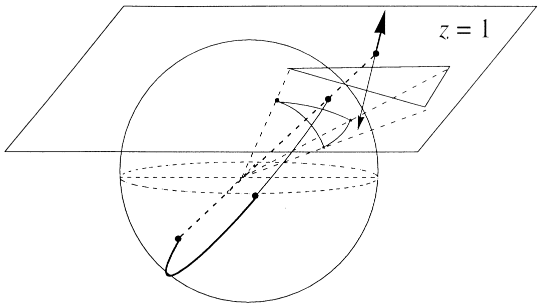

Fig. 6. Intersection of a semi-great circle and a spherical polygon under central projection.

polygons of opposite colors. (Refer to Ref. [1] for more details.) Therefore, a semi-great circle intersects a spherical polygon if and only if their images that are of the same color intersect, as illustrated in Fig. 6.

Let R and $\bar{R}$ correspond to the white and black half-lines (or rays) resulting from the central projection of a semi-great circle. Let $\omega(R)$ and $\beta(\bar{R})$ denote the respective number of white and black polygons intersected by ray R and $\bar{R}$. The *cutting weight* of this pair of opposite rays R and $\bar{R}$ is defined as $\omega(R)+\beta(\bar{R})-\phi(R,\bar{R})$, where $\phi(R,\bar{R})$ is the number of pairs of polygons—each pair being the image of a spherical polygon—which have its white and black images intersected by R and $\bar{R}$, respectively, as illustrated in Fig. 7. Rays R and $\bar{R}$ are called a maximal weight cutting pair if their cutting weight is a maximum. It can be verified easily that a semi-great circle intersects the maximal weight of spherical polygons if and only if its image is a maximal weight cutting pair with respect to the image of the spherical polygons.

As the problem of finding the maximal weight cutting pair can be reformulated into a more structural form in the dual domain, such a geometric duality is now described. Geometric duality in the plane is a transformation between the points and lines in the primal plane and its dual plane [9,10]. Let the axes in the primal plane be denoted as x- and y-axes, and the dual plane ξ- and η-axes. Under the duality, a homogeneous point $(a, b, 1)$ in the primal plane, such that $a \neq 0$ or $b \neq 0$, is mapped to a line $a\xi + b\eta + 1 = 0$ in the dual plane. Conversely, a homogeneous point $(\alpha, \beta, 1)$ (not at the origin) in the dual plane has its image in the primal plane as a line: $\alpha x + \beta y + 1 = 0$. Let the mappings from the primal to the dual and from the dual to the primal be denoted as Δ and Δ^{-1}, respectively. It is assumed that the lines in the primal plane are in general position, i.e., they do not pass through the origin and neither are they parallel to x- or y-axis. Such a setting can always be achieved in linear time by perturbing the input systematically.

With the point–line duality, the dual of a line segment can also be established. Let the line segment of interest be denoted as $\overline{uv}$ and the straight line coincident with $\overline{uv}$ be noted as L. Since $\overline{uv}$ consists of a continuous set of points on L, they are mapped continuously into lines in the dual plane that pass through a fixed point that is the dual of L. Thus, the dual of $\overline{uv}$ is a double-wedge whose apex is the dual

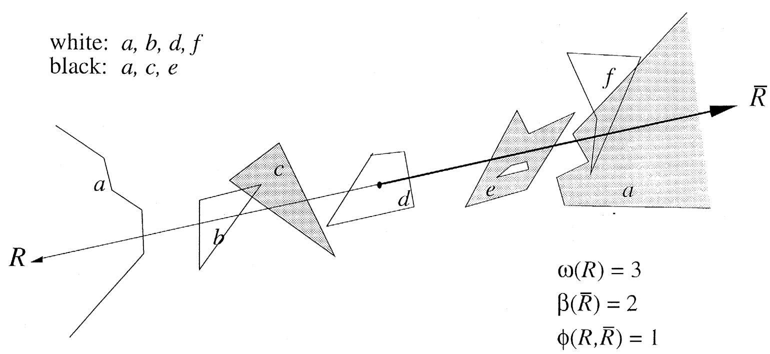

Fig. 7. Cutting weight of two opposite rays R and $\bar{R}$.

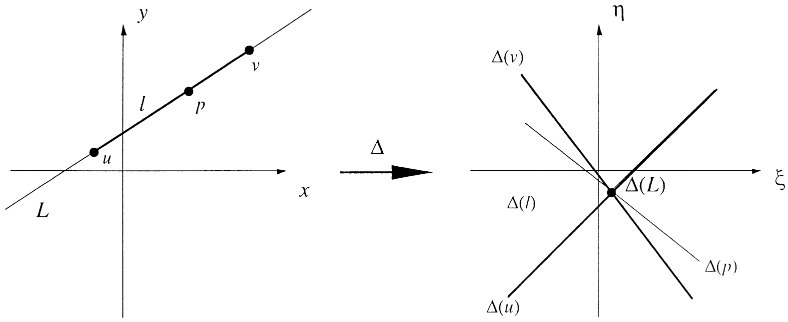

Fig. 8. Duality between a line segment and a double-wedge.

of L and whose two bounding lines are the dual of the two endpoints of $\overline{uv}$, as shown in Fig. 8. Notice that, because lines in the primal plane are assumed not to pass through the origin, the dual double-wedge $\Delta(\overline{uv})$ will not cover the origin of the dual plane. This property is, however, stretched to the extreme when $\overline{uv}$ becomes a half-line, as established in the following lemma.

Lemma 2.1. The dual of a half-line is a double-wedge with one of its bounding line passing through the origin of the dual plane.

Proof. Let $y = ax + b$ be the equation of the line containing the half-line such that a, $b \neq 0$ (since lines are assumed to be in general position). For a point $(x,\ ax+b)$ in the primal plane, its dual is a line $x\xi + (ax+b)\eta + 1 = 0$, which can be rewritten as $\eta = (-x/(ax+b))\xi - 1/x$. When x approaches $+\infty$ or $-\infty$, which corresponds to the endpoint of the half-line at infinity, this equation of the dual line becomes $\eta = (-1/a)\xi$. Q.E.D.

Through a simple analysis, it can also be shown that the dual of a double-wedge is either a line segment when the origin of the dual plane lies outside of the double-wedge, or a half-line when the origin lies on the boundary of the double-wedge. Therefore, a bijective correspondence between line segments and half-lines in the primal plane and those double-wedges whose interior do not cover the origin in the dual plane is established. It can be shown that the dual of the complement of $\Delta(\overline{uv})$ equals $(L - \overline{uv})$. This mapping nonetheless can be ignored as the input in the primal plane are restricted to individual line segments and half-lines. Thus, now and after, the double-wedges considered will only be those whose interior do not cover the origin, unless otherwise specified.

By the bijection between the line segments and half-lines and the double-wedges, the intersection between line segments and half-lines can be characterized by the geometric relation between their corresponding double-wedges in the dual plane. The lemma below gives such intersection relation in the dual space [11] with the proof omitted.

Lemma 2.2. A line segment or half-line L intersects another line segment or half-line L' if and only if $\Delta(L)$ and $\Delta(L')$ cover each other's apex.

Before proceeding to the next, the definition of a conjugate pair is introduced. Two non-parallel lines in the dual plane define two double-wedges which are complement to each other and are called a conjugate pair. For instance, the two double-wedges, one being plain and the other being shaded, defined by $\Delta(u)$ and $\Delta(v)$ in Fig. 8 constitute a conjugate pair. The dual of two opposite half-lines is thus a conjugate pair with one separating line passing through the origin of the dual plane, and will be called an $\mathcal{H}$-conjugate pair. Of an $\mathcal{H}$-conjugate pair, the separating line that does not pass through the origin is defined by the endpoint of the two half-lines and will be called the principal separating line; the other separating line would pass through the origin and the dual point of the line that coincides with the two half-lines in the primal, and will be called an auxiliary separating line. Note that, an $\mathcal{H}$-conjugate pair can be uniquely defined by its apex and the principal separating line.

3. Maximal weight cutting of line segments

This solves the following problem: given m white line segments $\{w_1, w_2, \dots, w_m\}$ and n black line segments $\{b_1, b_2, \dots, b_n\}$, find a ray R such that the cutting weight $\omega(R)+\beta(\bar{R})-\phi(R,\bar{R})$ is maximized.[4] The line segments are again assumed to be in general position. As indicated earlier, this requirement can be met easily by performing a simple geometric transformation on the input. It is first assumed that two polygons of different colors do not share a same identifier, and consequently the effect of $\phi(R,\bar{R})$ is ignored since it always equals zero. The case that $\phi(R,\bar{R}) \neq 0$ will be handled at a later stage.

Let R be a ray in the plane such that R intersects white line segments $\{w_1, w_2, \dots, w_j\}$ and its opposite ray $\bar{R}$ intersects black line segments $\{b_1, b_2, \dots, b_k\}$. A double-wedge $\mathscr{W}$ is said to cover another double-edge $\mathscr{W}'$ if $\mathscr{W}$ contains the apex of $\mathscr{W}'$. By Lemma 2.2, $\Delta(R)$ covers $\Delta(w_i)$, for $1 \le i \le j$, and $\Delta(\bar{R})$ covers $\Delta(b_i)$, for $1 \le i \le k$. Conversely, given a $\mathscr{H}$-conjugate pair in the dual plane, such that one covers $\{\Delta(w_1), \Delta(w_2), \dots, \Delta(w_j)\}$ and the other covers $\{\Delta(b_1), \Delta(b_2), \dots, \Delta(b_k)\}$, there must exist two opposite rays R and $\bar{R}$ in the primal plane such that $\omega(R)=j$ and $\beta(\bar{R})=k$. To describe this one-to-one relationship concisely, a few more terms for a conjugate pair are needed.

Let $\mathscr{W}_{\mathscr{H}}$ denotes an $\mathscr{H}$-conjugate pair. The double-wedge of $\mathscr{W}_{\mathscr{H}}$, resulting from the clockwise rotation about the apex from the auxiliary separating line to the principal separating line, is called the CW component of $\mathscr{W}_{\mathscr{H}}$ and is denoted as $\mathscr{W}^+_{\mathscr{H}}$. The other double-wedge of $\mathscr{W}_{\mathscr{H}}$ is called the CCW component of $\mathscr{W}_{\mathscr{H}}$ and is denoted as $\mathscr{W}^-_{\mathscr{H}}$. See Fig. 9a. Let $\omega(\mathscr{W}^+_{\mathscr{H}})$ and $\beta(\mathscr{W}^+_{\mathscr{H}})$ denote the white and black weight of double-wedge $\mathscr{W}^+_{\mathscr{H}}$, respectively, which are also equal to the number of white and black line segments whose duals are covered by $\mathscr{W}^+_{\mathscr{H}}$. Analogous definitions can be established for the double-wedge $\mathscr{W}^-_{\mathscr{H}}$.

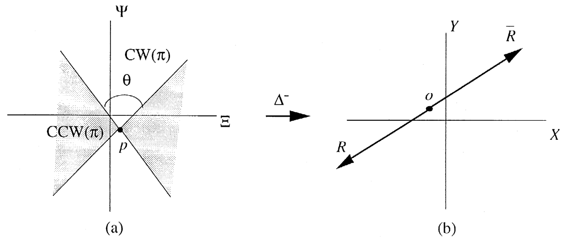

Fig. 9. CW and CCW components of an $\mathscr{H}$-conjugate pair.

Let R and $\bar{R}$ be the $\Delta^{-1}(\mathscr{W}^+_{\mathscr{H}})$ and $\Delta^{-1}(\mathscr{W}^-_{\mathscr{H}})$ respectively, see Fig. 9b. The two numbers $\omega(\mathscr{W}^+_{\mathscr{H}}) + \beta(\mathscr{W}^-_{\mathscr{H}})$ and $\beta(\mathscr{W}^+_{\mathscr{H}}) + \omega(\mathscr{W}^-_{\mathscr{H}})$ determine the color of the segments that R and $\bar{R}$ will intersect. The choice of the number $\omega(\mathscr{W}^+_{\mathscr{H}}) + \beta(\mathscr{W}^-_{\mathscr{H}})$ indicates that R will intersect white line segments and $\bar{R}$ black line segments, whereas the choice of the number $\beta(\mathscr{W}^+_{\mathscr{H}}) + \omega(\mathscr{W}^-_{\mathscr{H}})$ yields the opposite. The number $\max\{\omega(\mathscr{W}^+_{\mathscr{H}}) + \beta(\mathscr{W}^-_{\mathscr{H}}), \beta(\mathscr{W}^+_{\mathscr{H}}) + \omega(\mathscr{W}^-_{\mathscr{H}})\}$ will be referred to be the *covering weight* of $\mathscr{W}_{\mathscr{H}}$. The maximal weight cutting of line segments can then be defined in the dual space as follows.

Problem. Maximal weight cutting of line segments. Given a set of line segments, find, in the dual space, an $\mathscr{H}$-conjugate pair $\mathscr{W}_{\mathscr{H}}$ such that the covering weight of $\mathscr{W}_{\mathscr{H}}$, $\max\{\omega(\mathscr{W}^+_{\mathscr{H}}) + \beta(\mathscr{W}^-_{\mathscr{H}}), \beta(\mathscr{W}^+_{\mathscr{H}}) + \omega(\mathscr{W}^-_{\mathscr{H}})\}$, is maximized.

An $\mathscr{H}$-conjugate pair $\mathscr{W}_{\mathscr{H}}$ can be uniquely defined by two parameters: its apex p, and the angle θ of its CW component $\mathscr{W}^+_{\mathscr{H}}$, as illustrated in Fig. 9a. In other words, an $\mathscr{H}$-conjugate pair $\mathscr{W}_{\mathscr{H}}$ can be denoted as $\mathscr{W}_{\mathscr{H}}(p,\theta)$, a function of p and θ. For a given apex p, the set $\{\mathscr{W}_{\mathscr{H}}(p,\theta)\colon 0° \le \theta \le 180°\}$ gives a family of $\mathscr{H}$-conjugate pairs whose duals are contained by the same line. Geometrically, when θ varies from 0° to 180°, the endpoint s of the two opposite rays $\Delta^{-1}(\mathscr{W}_{\mathscr{H}}(p,\theta))$ moves from one end (at infinity) of the line $\Delta^{-1}(p)$ to its other end (at infinity). (Refer to Fig. 9.) It will be shown later that, for a fixed point p, $O(e \log e)$ time suffices to finding a θ such that the $\mathscr{H}$-conjugate pair $\mathscr{W}_{\mathscr{H}}(p,\theta)$ maximizes its covering weight, where e is the number of line segments that the line $\Delta^{-1}(p)$ intersects.

[4] For convenience, a half-line will also be called a line segment.

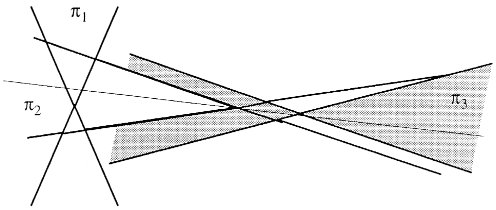

Fig. 10. The arrangement of three double-wedges.

Let $\lambda(p)$ denote the maximum of the covering weights of the family of $\mathscr{H}$-conjugate pairs $\{\mathscr{W}_{\mathscr{H}}(p,\theta)$: $0° \le \theta \le 180°\}$. The maximization of cutting weight in the dual space can now be reformulated as follows.

Problem. (Revised) Maximal weight cutting of line segments. Find a point p in the dual plane that maximizes $\lambda(p)$.

It is to be shown that by grouping the points in the dual plane into *congruent sets*, only a discrete number of tests need to be performed to find an optimal p. Two points p and q are said to be congruent if $\lambda(p) = \lambda(q)$. If the number of congruent sets is finite, conceivably, the searching for p can be carried out in a discrete manner by considering only one point for each congruent set. Fortunately, it is indeed finite and is bounded by $O((m+n+I_{wb})^2)$, where m is the number of white line segments, n is the number of black line segments and I_{wb} is the number of pairwise intersections between a white and a black line segment. It is noted that the dual of the intersection between a white and a black line segment is a line that passes through the apexes of the dual double-wedges of these two line segments. Let $\mathscr{A}$ denote the arrangement of the $2(m+n)$ bounding lines of the $(m+n)$ double-wedges and the I_{wb} lines. [5] Fig. 10 shows such an arrangement with two white double-wedges $\mathscr{W}_1$ and $\mathscr{W}_2$ and a black double-wedge $\mathscr{W}_3$. This arrangement induces $O((m+n+I_{wb})^2)$ convex faces [12], each of which is a congruent set as established in the following theorem.

[5] An arrangement in the plane is a partition of the plane by a set of lines.

Theorem 3.1. Every induced face in the arrangement $\mathscr{A}$ is a congruent set.

Proof. Let p be an interior point in a face of $\mathscr{A}$ and t be an arbitrary point on the boundary of that face. Let $\mathscr{W}_{\mathscr{H}}$ be a $\mathscr{H}$-conjugate pair that realizes $\lambda(p)$ and, without loss of generality, suppose $\mathscr{W}^+_{\mathscr{H}}$ covers white double-wedges $\{\Delta(w_1), \Delta(w_2), \ldots, \Delta(w_h)\}$ and $\mathscr{W}^-_{\mathscr{H}}$ covers black double-wedges $\{\Delta(b_1), \Delta(b_2), \ldots, \mathscr{T}(b_k)\}$, with $\lambda(p) = \omega(\mathscr{W}^+_{\mathscr{H}}) + \beta(\mathscr{W}^-_{\mathscr{H}})$. It is shown that for any point $q \in (p,t]$, there is always an $\mathscr{H}$-conjugate pair $\mathscr{W}_{\mathscr{H}}(q,\theta)$ for some Θ such that $\mathscr{W}^+_{\mathscr{H}}$ covers $\{\Delta(w_1), (w_2), \ldots, \Delta(w_h)\}$ and $\mathscr{W}^-_{\mathscr{H}}$ covers $\{\Delta(b_1), \Delta(b_2), \ldots, \Delta(b_k)\}$. Thus, $\lambda(p) \le \lambda(q)$. The theorem is then readily proven by the arbitrariness of p and t.

The proof is carried out by providing a moving strategy for the point q from p toward t. When initially moving q from p toward t, the angle θ of the conjugate pair $\mathscr{W}_{\mathscr{H}}(q,\theta)$ is chosen in the way that its principal separating line is always parallel to that of p. This movement continues until some $\Delta(w_i)$ is just about to leave $\mathscr{W}^+_{\mathscr{H}}$ or a $\Delta(b_j)$ is just about to leave $\mathscr{W}^-_{\mathscr{H}}$. Suppose $\Delta(w_i)$ is just about to leave $\mathscr{W}^+_{\mathscr{H}}$. Because that a double-wedge is a connected point set and the motion of the principal separating line of $\mathscr{W}_{\mathscr{H}}$ is continuous, this happens only when the apex of $\Delta(w_i)$ is on one separating line of $\mathscr{W}_{\mathscr{H}}$. Only two cases are possible:

(i) the apex of $\Delta(w_i)$ is on the auxiliary separating line of $\mathscr{W}_{\mathscr{H}}$, or

(ii) the apex of $\Delta(w_i)$ is on the principal separating line of $\mathscr{W}_{\mathscr{H}}$.

However, case (i) is impossible, unless q is already the point t, as shown in Fig. 11a. Otherwise, the double-wedge $\Delta(w_i)$ would strictly cover the origin of the dual plane, conflicting our assumption that w_i is a line segment or half-line, see Fig. 11b.

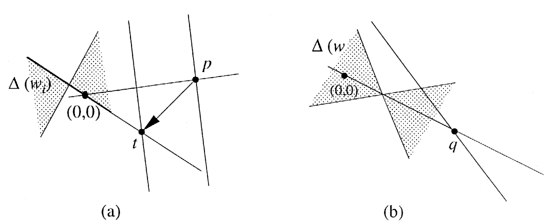

Fig. 11. Proof of Theorem 3.1, case (i).

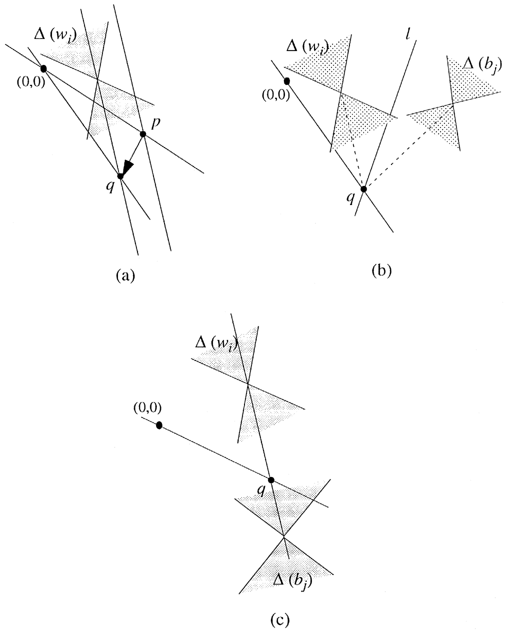

Fig. 12. Proof of Theorem 3.1, case (ii).

Fig. 12a illustrates case (ii). The principal separating line of $\mathscr{W}_{\mathscr{H}}$ is then rotated clockwise about the apex q by an angle $\alpha/2$, where α is the sweeping angle incurred when rotating the current principal line clockwise until it hits an apex of some black double-wedge $\Delta(b_j)$ (e.g., the line l in Fig. 12b). If α is not zero, $\Delta(w_i)$ is now strictly covered by $\mathscr{W}^{+}_{\mathscr{H}}$, and q is moved toward t just as it was moved when starting from p. The situation that $\alpha = 0$ can only happen when q is on a line passing through the apexes of two mutually covered white and black double-wedges, see Fig. 12c, in which q is already the endpoint t.

It is a simple matter to devise a similar treatment when a $\Delta(b_j)$ is just about to leave $\mathscr{W}^{-}_{\mathscr{H}}$. Therefore, through this 'keeping parallel' and 'rotating to cover' updating of the angle Θ of $\mathscr{W}_{\mathscr{H}}(q,\Theta)$, when q finally reaches t, it is guaranteed that $\{\Delta(w_1), \Delta(w_2), \dots, \Delta(w_h)\}$ and $\{\Delta(b_1), \Delta(b_2), \dots, \Delta(b_k)$ are covered by $\mathscr{W}^{+}_{\mathscr{H}}$ and $\mathscr{W}^{-}_{\mathscr{H}}$ respectively, i.e., $\lambda(p) \leq \lambda(q)$. Q.E.D.

The remaining of this is devoted to establish an algorithm that finds a maximal weight cutting pair R and $\overline{R}$ of a set of white and black line segments in $O((E + I_{wb})^2 E)$ time, where E is the total number of line segments. To pursue this algorithm, the following two observations are made: i) for a point p in a face, the set of double-wedges covered by the $\mathscr{H}$-conjugate pair $\mathscr{W}_{\mathscr{H}}(p,\Theta)$ is the same for any $\theta \in [0°,180°]$; and ii) for any two interior points p and q in a face, $\mathscr{W}_{\mathscr{H}}(p,\theta)$ and $\mathscr{W}_{\mathscr{H}}(q,\theta')$ cover the same set of double-wedges with any θ, $\theta' \in [0°,180°]$.

The first observation becomes apparent if one notes that the dual of the point p in the dual plane is a line in the primal plane which intersects a fixed set of line segments. The second observation can be verified by the fact that when a $\mathscr{W}_{\mathscr{H}}(p,\Theta)$ is about to cover or not to cover a double-wedge, its apex must be on one separating line of that double-wedge, i.e., on the boundary of a face. Calling this set of double-wedges the *covering set* of that face, which is invariant to the points in the face, an ordering is imposed next on it.

Let p be a point in a face of the arrangement $\mathscr{A}$. The sequence of the colors of the double-wedges in the covering set of p, incurred when being scanned by a line through p and the origin rotating clockwise about p for 180°, is called the *characteristic sequence* at p.

Fig. 13 illustrates an example of a characteristic sequence. The covering set of the face containing p has five double-wedges with the apexes p_1, p_2, p_3, p_4, and p_5, respectively. Letting $C(p_i)$ denote the color of the double-wedge with the apex p_i, the characteristic sequence at p is then $\{C(p_2), C(p_3), C(p_1), C(p_5), C(p_4)\}$. For instance, if the three double-wedges with the apexes p_2, p_3 and p_4 are black and the other two are white, the characteristic sequence substantiates to {'black', 'black', 'white', 'white', 'black'}.

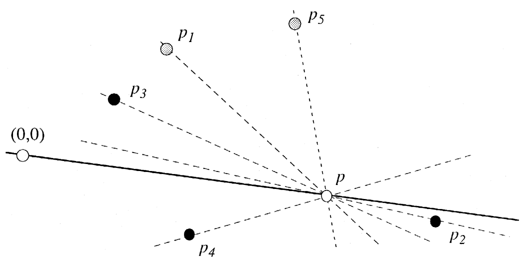

Fig. 13. Characteristic sequence at a point p.

A sub-sequence in a characteristic sequence is called a white maximal clustering if all the elements in it are white, and its preceding and succeeding elements are black (unless there is no preceding or succeeding element). Analogous definition applies to a black maximal clustering. The number of the elements in a white or black maximal clustering is then called the weight of that clustering. A characteristic sequence is said to be *homogenized* if all the elements in a maximal clustering are replaced with a single pair of the color and weight of that clustering. In the example shown in Fig. 13, if p_2, p_3, and p_4 are black and p_1 and p_5 are white, the homogenized characteristic sequence is then a sequence of three pairs {('black', 2), ('white', 2), ('black', 1)}.

It is a simple matter to conceive that, given the covering set of a point p with e double-wedges, the characteristic sequence can be constructed in $O(e \log e)$ time, using any quicksort algorithm [13]. The homogenized characteristic sequence as well as $\lambda(p)$ can then be obtained in $O(e)$ time by a linear scanning of the characteristic sequence. The crux for improving the $O(e \log e)$ time complexity resides on the following idea: instead of being constructed from scratch, the homogenized characteristic sequence at a point of a face is derived by updating that of its neighboring faces. If this updating can be done in linear time (with the cardinality of the sequence), through a propagation of the faces of the arrangement $\mathscr{A}$, an $O((E+I_{wb})^2 E)$ time algorithm becomes conceivable, where E is the total number of line segments. Before such a linear updating scheme is given, it is necessary to show that all the points in a face of $\mathscr{A}$ have the same homogenized characteristic sequence.

Lemma 3.2. Any two points p and q in a face of $\mathscr{A}$ have the same homogenized characteristic sequence.

Proof. Suppose the covering set of the face containing p and q has k double-wedges and their apexes are $p_1, p_2, \ldots, p_k$. Define a continuous angular function $\alpha(t,p_i)$ as shown in Fig. 14a, where t moves from p to q along the line segment $[p,q]$. Without loss of generality, assume the characteristic sequence at p to be $\{C(p_1), C(p_2), \ldots, C(p_k)\}$, and $\{C(p'_1), C(p'_2), \ldots, C(p'_k)\}$ at q, where $\{p'_1, p'_2, \ldots, p'_k\}$ is a permutation of $p_1, p_2, \ldots, p_k$. The proof is carried out in two phases.

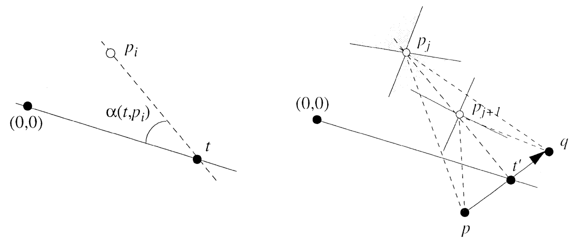

Fig. 14. Proof of Lemma 3.2.

Phase 1. Suppose every maximal clustering at p is composed of only one element. It is claimed that p'_i must be p_i, for any $1 \le i \le k$. For if not, because of the continuity of $\alpha(t,p_i)$ $(1 \le i \le k)$ and the initial ordering $\alpha(t,p_1) < \alpha(t,p_2) < \ldots < \alpha(t,p_k)$, there must exist a point t' between p and q at which $\alpha(t',p_j)$ identifies with $\alpha(t',p_{j+1})$ for some $1 \le j < k$. Since, by the definition of a maximal clustering, every pair of p_j and p_{j+1} $(1 \le j < k)$ has a different color, this can only happen when the two points p_j and p_{j+1} of different colors, and the point t' are collinear, see Fig. 14b.

Phase 2. Assume some maximal clusterings in the characteristic sequence at p have more than one element. Construct a restricted characteristic sequence S' at p by only selecting one element from each maximal clustering at p. By the proof of phase 1, when t moves from p to q, the corresponding restricted characteristic sequence remains the same as S'. Due to the arbitrariness of the pairs selected from the maximal clusterings when forming S', it follows that for any apex p_{r+i} in a maximal clustering $M = \{C(p_r), C(p_{r+1}), \ldots, C(p_{r+h})\}$ at p, $0 \le i \le h$, the inequality $\alpha(q,p_s) < \alpha(q,p_{r+i}) < \alpha(q,p_v)$ is always true, where p_s and p_v are any apexes in the maximal clustering immediately before and after M respectively. Since M can be any maximal clustering at p, the only conclusion is that $\{p'_r, p_{r+1}, \ldots, p'_{r+h}\}$ is a permutation of $p_r, p_{r+1}, \ldots, p_{r+h}$. Q.E.D.

To facilitate the updating of a homogenized characteristic sequence, a double-linked data list is used to store the sequence. Each maximal clustering is represented as a node in the list with four fields: its color, its weight, the front pointer that points to its preceding maximal clustering node, and the rear pointer pointing to its succeeding maximal clustering node. An additional pointer HCS is provided to specify the first maximal clustering node in the list.

Let each double-wedge in the dual plane be adhered a pointer called *index pointer*: it points to some unique maximal clustering node in the HCS list if this double-wedge is included in that clustering, or is set to nil otherwise. By Lemma 3.2, this index pointer is invariant for a face of $\mathscr{A}$. Let p and q be two points in two neighboring faces of $\mathscr{A}$, respectively. To obtain the HCS list as well as the corresponding index pointers at point q from that at p, the following updates are classified based on the type of the line that separates the two faces.

(i) The separating line is a separating line of a double-wedge which covers p.

This double-wedge should be deleted from the HCS list at q since it does not cover q. To reflect this deletion, the node in the HCS list that pertains to the double-wedge is fetched (in constant time) via the index pointer of the double-wedge. The weight of this node is then decremented by 1. The node itself will be deleted from the HCS list if its reduced weight is zero. The index pointer of the double-wedge is then set to nil. The entire operation takes constant time.

(ii) The separating line is a separating line of a double-wedge which does not cover p.

This double-wedge ought to be added to the HCS list at q since it now covers q. This requires first to locate the two adjacent nodes between which the double-wedge should be inserted. Let $p_1, p_2, \dots, p_k$ be the apexes of the covering set at p, which can be identified in linear time through a scanning of the index pointers of all the double-wedges. A pair p_r and p_s are then identified in $O(k)$ time such that $\alpha(q,p_r)$ is the greatest among those $\alpha(q,p_i)$ that are smaller than $\alpha(q,p')$, and $\alpha(q,p_s)$ is the smallest among those $\alpha(q,p_i)$ that are greater than $\alpha(q,p')$, where p' is the apex of the double-wedge to be added. (Refer to Fig. 14a for the definition of $\alpha(q,p_i)$.) If p_r and p_s are of different colors, we increment by 1 the weight field of the node pointed to by one of the index pointers of p_r and p_s that has the same color as p', and set the index pointer of p' to this node. If p_r, p_s and p' all have the same color, the treatment is virtually the same except this time both p_r and p_s are in a same node. Finally, suppose p_r and p_s are of the same color, say white, but p' is black. The node in the HCS list that pertains to p_r and p_s must then be split into three nodes to reflect the insertion of p'. That which new white node should a double-wedge of the original white node belongs to can be determined by comparing its angle with that of p'. Specifically, a double-wedge p_i of the original white node is included by the new white node in front (back) of the newly inserted black node if and only if $\alpha(q,p_i)$ is less (greater) than $\alpha(q,p')$. This classification can certainly be done in time linear to the number of double-wedges of the node to split.

(iii) The separating line passes through the apexes of two mutually covered white and black double-wedges.

This corresponds to the case shown in Fig. 14b. Let p_w and p_b be the apexes of the white and black double-wedges, and M_w and M_b be the maximal clusterings at p that p_w and p_b belong to respectively. Without loss of generality, suppose M_w is in front of M_b. To reflect the ordering changing of $\alpha(q,p_w)$ and $\alpha(q,p_b)$, these two maximal clusterings must be split into four consecutive maximal clusterings, M'_w, p_b, p_w, M'_b. With reasoning similar to (b), this split as well as the updating of index pointers can be done in linear time.

The revised algorithm for computing the maximal weight cutting pairs is now outlined as follows. Let Γ be the dual of $\mathscr{A}$ and $\psi(\Gamma)$ the minimum spanning tree [13] of Γ. A vertex in the dual Γ corresponds to a face of $\mathscr{A}$, and two vertices are connected by an edge if the two faces they represent are neighbors in $\mathscr{A}$. With Lemma 3.2, each vertex is associated with a unique homogenized characteristic sequence and the index pointers set of the double-wedges. This homogenized characteristic sequence as well as those index pointers are to be obtained for every face in $\mathscr{A}$ through a traversal of the tree $\psi(\Gamma)$. Specifically, the homogenized characteristic sequence HCS list as well as the corresponding index pointers at the root of $\psi(\Gamma)$ is computed in $O(E \log E)$ time, and the cutting weight corresponding to this homogenized characteristic sequence is recorded in a global variable, say MAX_CW. During the traversal of $\psi(\Gamma)$, when a vertex v is being visited, the HCS list as well as the index pointers is updated in linear time according to the prescribed treatment on that of the immediately previously visited vertex which is connected with υ by an edge. The covering weight $\lambda(\upsilon)$ [6] is then calculated, also in linear time, based on the updated HCS list and the index pointers, and is assigned to MAX_CW if it is the greater. This traversal results in the maximal cutting weight MAX_CW and a corresponding vertex υ that realizes it. It is not hard to see that the total number of visits to the vertices is two times the number of edges in $\psi(\Gamma)$, which is bounded by $O((E+I_{wb})^2)$. Since the minimal spanning tree of a graph can be established in linear time with the size of the graph [13], the following result is in order.

Theorem 3.3. All the maximal weight cutting pairs of E white and black line segments can be found in $O((E+I_{wb})^2E)$ time and $O((E+I_{wb})^2)$ space.

Before leaving this, it is yet necessary to resolve the problem when two differently colored line segments have the same identifier. Let us first re-color the line segments as follows: a line segment retains its original color if its identifier is not shared by any other line segment of an opposite color, or is painted gray otherwise. Based on this classification, the cutting weight of two opposite rays R and $\overline{R}$ is easily seen to be $\omega(R)+\beta(\overline{R})+\gamma(R,\overline{R})$, where $\gamma(R,\overline{R})$ indicates the number of distinct gray line segments intersected by the line collinear with R and $\overline{R}$. (Two line segments are distinct if their identifiers are different.)

Let $\mathscr{A}$ be the arrangement of the bounding lines of the double-wedges which are the dual of the given line segments, and the lines which are the dual of the intersection points of the line segments of opposite colors. To modify the prescribed revised algorithm to accommodate gray line segments, a register is associated with each distinct identifier of gray line segments. Initially, all the registers are reset to zero. Let s be the line segment whose dual contributes to the separating line between two adjacent faces f and f' in the tree $\psi(\Gamma)$. When traversing from f to f', the updating action is taken according to the color of s. If s is not gray, the HCS list, MAX_CW and index pointers are updated the same way as in the revised algorithm. Otherwise, the register of the identifier of s is either incremented or decremented by 1, depending on whether $\Delta(s)$ does or does not cover f'. The variable MAX_CW itself is then incremented or decremented by 1 if the updated register holds a value 1 or 0. That MAX_CW changes only corresponding to 0 and 1 values of the relevant register prevents possible multiple counting of non-distinct gray line segments, and hence ensures the correct cutting weight. As this additional updating of the register and variable MAX_CW only takes constant time, it is concluded that Theorem 3.2 still holds.

4. Maximal weight cutting of polygons

With the result of Section 3 in hand, the maximal weight cutting problem for a set of white and black polygons $\{f_1, f_2, \dots, f_N\}$ can be solved accordingly. Let the edges of a polygon f_i have the color and identifier of f_i. The edges of all the polygons along with the intersections of the edges of opposite colors thus induce an arrangement $\mathscr{A}$ in the dual plane, as described in Section 3. Akin to $\lambda(p)$, the function $\Lambda(p)$ of p in the dual plane is defined to be the maximal cutting weight of polygons that two opposite rays R and $\overline{R}$ can realize, with R and $\overline{R}$ restricted to be collinear with the line $\Delta^{-1}(p)$. With the same spirit, two points p and q in the dual plane are said to be congruent to each other if $\Lambda(p)=\Lambda(q)$. It is to be examined how can a $\Lambda(p)$ be calculated based on the characteristic sequence (of the edges) at p. For simplicity of discussion, we will assume that there is not any gray edge, as the register method works regardless of the number of gray line segments that share a same identifier.

Suppose the dual line $\Delta^{-1}(p)$ intersects a polygon f_i and, without loss of generality, let $\{p_1, p_2, \dots, p_k\}$ be the clockwise ordered apexes of the double-wedges of those edges of f_i intersected by $\Delta^{-1}(p)$. Refer to Fig. 15, p_1 and p_k are called the *heading* and *tailing* apexes of f_i respectively. (Note that k could be 1 if f_i is unbounded, in which p_k is both the heading and tailing apex.) If the CW component of an $\mathscr{H}$-conjugate pair $\mathscr{W}_{\mathscr{H}}(p,\Theta)$ covers

[6] For simplicity of notation, $\lambda(\upsilon)$ is used to mean $\lambda(p)$ where p is any point in the face of Γ represented by the vertex υ.

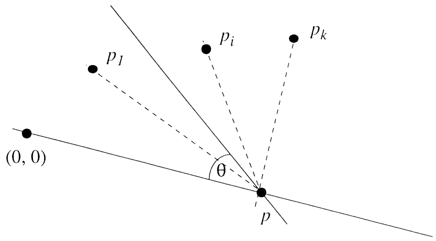

Fig. 15. Heading and tailing apexes of a polygon.

some p_i, it must also cover p_1; and the CCW component of $\mathscr{W}_{\mathscr{H}}(p,\Theta)$ covers some p_i only if it also covers p_k. This suggests that the $\{p_2, p_3, \dots, p_{k-1}\}$ can be deleted from the characteristic sequence, since their covering information, being CW or CCW covering, is always implied by the two extreme points p_1 and p_k. The characteristic sequence at p after such a deletion operation for every relevant polygon will be said to have been *relaxed*.

The $\lambda(p)$ however may not be equal to the $\Lambda(p)$ even when the characteristic sequence is relaxed, as the weight of a polygon f_i might have been double counted. This discrepancy can be remedied as follows. When searching for the θ that realizes $\max\{\omega(\mathscr{W}_{\mathscr{H}}^{+}(p,\theta)) + \beta(\mathscr{W}_{\mathscr{H}}^{-}(p,\theta))$: $0° \leq \theta \leq 180°\}$, the tailing apexes of white polygons and the heading apexes of black polygons should be ignored. This is because the coverage of the double-wedges of a white polygon by a $\mathscr{W}_{\mathscr{H}}^{+}(p,\Theta)$ is uniquely determined by the coverage of its heading apex, whereas a $\mathscr{W}_{\mathscr{H}}^{-}(p,\Theta)$ covers some double-wedges of a black polygon if and only if it covers the tailing apex of that polygon. With the same reasoning, when computing $\max\{\beta(\mathscr{W}_{\mathscr{H}}^{+}(p,\theta)) + \omega(\mathscr{W}_{\mathscr{H}}^{-}(p,\theta))$: $0° \leq \theta \leq 180°\}$, only the tailing apexes of white polygons and the heading apexes of black polygons will be considered. Therefore, through this further relaxation, the resultant $\lambda(p)$ is guaranteed to be $\Lambda(p)$.

The relationship between $\Lambda(p)$ and $\mathscr{A}$ is fully manifested by the observation that each face in $\mathscr{A}$ is also a congruent set under the Λ measure. To see this, let $M = \{M_1, M_2, \dots, M_k\}$ be the homogenized and relaxed characteristic sequences at a point p. Using the induction similar to the proof of Lemma 3.1 and recalling that all the edges of a polygon have a same color, it can be shown that:

(i) the homogenized and relaxed characteristic sequences at any point of the face of p is equal to M, and
(ii) if the heading or tailing apex of a f_i is in some maximal clustering M_s at p, it must also appear in the M_s at any point of the face of p.

It then follows that if two points p and q have their homogenized and relaxed characteristic sequences satisfy the above two conditions, $\Lambda(p)$ must be equal to $\Lambda(q)$.

To implement the relaxation when traversing the tree $\psi(\Gamma)$, in addition to the (relaxed) HCS list and index pointers of edges as defined in the previous, two identifiers, HEAD and TAIL, are associated with a polygon f_i; they specify the edges whose duals are the heading and tailing double-wedges of f_i in the homogenized and relaxed characteristic sequence. When traversing from a point p to another point q, each belonging to a neighboring face in Γ, similar actions are taken just as that in Section 3. In addition, some 'house-keeping' work must be performed in order to keep the relaxation of the HCS list. For example, when adding (the double-wedge of) an edge e_i into the HCS list, one must first check the HEAD and TAIL of the polygon which e_i belongs to. If neither one of them is defined, e_i is added to HCS list and the HEAD and TAIL are both set to point to e_i. Otherwise, e_i is added only when its α angle at q is less than that of the HEAD edge or greater than that of the TAIL edge. If e_i is added, say as a HEAD edge, the HEAD pointer must be alternated to reflect this change and, if the old HEAD and TAIL are not the same, the old HEAD edge should be deleted from the HCS list. We omit these details but only state that all these 'house-keeping' work can be done in constant time.

The naive upper-bound of the time required by a traversal on the tree $\psi(\Gamma)$ is $O((E+I_{wb})^2 E)$, as indicated by Theorem 3.3, where E is the total number of edges. This bound can however be reduced to $O((E+I_{wb})^2 N)$, via a list called *inclusion list*. This list contains the pointers to those edges that are in the current HCS list. It is updated accordingly when an edge is added or deleted from the HCS list, and keeps its length to be no greater than $2N$. Such a list enables us to identify those edges of the HCS list in $O(N)$ time. Since an updating on the inclusion list takes $O(N)$ time and, as trivially conceivable, the

$\Lambda(p)$ can be computed in $O(N)$ time given the HCS list, the inclusion list, and the HEAD and TAIL pointers, the following result is achieved.

Theorem 4.1. The maximal weight cutting pairs of N polygons $\{f_1, f_2, \ldots, f_N\}$ with a total of E edges can be found in $O((E + I_{wb})^2 N)$ time and $O((E + I_{wb})^2)$ space, where I_{wb} is the number of intersections between the edges of polygons of different colors. In particular, if white polygons and black polygons are disjoint from each other, the problem is solvable in $O(E^2 N)$ time and $O(E^2)$ space.

Finally, it is remarked that our algorithm accommodates not necessarily polygons, but actually groups of line segments. More specifically, a f_i in Theorem 4.1 needs not to be a polygon at all. The only implication of f_i is that the multi-count of the intersections of its edges by a ray should be eliminated. It remains to be a challenging problem to see whether the properties of polygons, in particular convex polygons, can be taken advantage of to reduce the time bound $O((E + I_{wb})^2 N)$.

5. Summary

Unlike a 3-axis NC machine, a 4-axis machine offers an infinite number of tool axis for a single workpiece setup. This paper addresses the following optimization problem: how can a workpiece be set up on a 4-axis machine so that the maximal number of surfaces can be machined?

A polynomial time algorithm is proposed, invoking the concept of VMaps and spherical polygons. Given the VMaps of N sampling points on the part surface, represented as spherical polygons, it is shown that $O((E + I_{wb})^2 N)$ time suffices to finding a semi-great circle that intersects the maximal number of VMaps, where E is the total number of edges of VMaps and I_{wb} is the number of pairs of edges that are antipodal to each other. Such a semi-great circle corresponds to a rotational axis (fourth axis) with 180° allowable rotation angle, which conforms to most of today's 4-axis machines with a flat work table.

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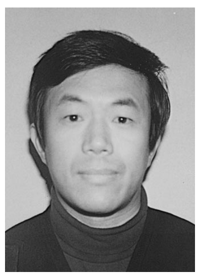

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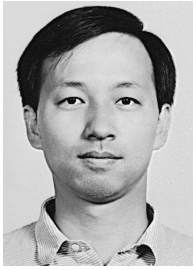

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