Unique Quantification of Critical State in Granular Media Considering Fabric Anisotropy

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Summary. Classic definition on the critical state of granular soils lacks proper reference to the fabric structure developed at critical state. This paper presents a study on the characteristics of critical state in consideration of fabric anisotropy in granular media, based on Discrete Element Method (DEM) simulations and a *contact-based* fabric tensor. In addition to the unique relation between the critical void ratio and the mean effective stress, we identify a unique relation between the mean effective stress and a fabric anisotropy parameter, K, defined by the first joint invariant of deviatoric stress tensor and the deviatoric fabric tensor. The relation does not depend on the specific loading paths. The two unique relations combine to present a unique spatial critical state line (CSL) in the 3Dl space K-e-p for a granular material whose projection in the e-p plane is the unique CSL in classic critical state theory.

Keywords: critical state, fabric tensor, anisotropy, spatial critical sate line, DEM.

1 Introduction

The classic critical state refers to an ultimate state of continuous shear deformation with constant volume under constant stress. This definition of critical state emphasizes only the scalar-valued void ratio (or fabric isotropy) and lacks a proper reference to the behavior of anisotropic fabric. Numerous studies have indicated that the behavior of granular soil is predominantly anisotropic under shear (see [1-3]). The fabric anisotropy has to be considered in the definition of critical state. Motivated by the Anisotropic Critical State Theory (ACST) recently proposed by Li & Dafalias (see [4]), the present study seeks to use DEM to explore the unique characteristics of critical state in consideration of fabric anisotropy.

2 Contact-Normal Based Fabric Tensor and Joint Invariants

In a cubic container with rigid walls we generate around 32,000 polydisperse spherical particles with radii ranging from 0.2 mm to 0.6 mm. A linear contact law

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with Coulomb's friction governing the sliding is employed. The interparticle friction coefficient adopts a value at 0.5. Isotropic samples with different initial void ratios are prepared using a technique proposed in Guo & Zhao (2012). The obtained samples are montonically sheared under either drained or undrained (constant volume) conditions. Special numerical schemes are designed to shear the samples at a constant intermediate principal stress ratio $b=(\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)$ loading path. Five cases of b have been investigated, $b=0,\ 0.25,\ 0.5,\ 0.75,\ 1$. Constant p' tests are also carried out to obtain more data points. We employ the ***contact-based*** definition of fabric tensor proposed by Satake (see [5]) to characterize the fabric structure:

$$F_{ij}=15(\phi_{ij}-\delta_{ij}/3)/2, \quad \phi_{ij}=\int_{\Theta}E(\Theta)n_i n_j d\Theta \tag{1}$$

where **n** is the unit normal vector between two contacted particles. Θ denotes the orientation of **n** relative to the global coordination system. $E(\Theta)$ is the distribution probability density function (PDF). The second-order Fourier expansion of $E(\Theta)$ is adopted to characterize the contact normals such that $E(\Theta)=(1+F_{ij}n_i n_j)/(4\pi)$. The second-order fabric tensor F_{ij} is deviatoric and symmetric, and will be used to characterize the fabric anisotropy in an assembly. Its invariants and joint invariants with the deviatoric stress tensor are defined below

$$J_1^F=F_{ii}=0,\ J_2^F=\frac{1}{2}F_{ij}F_{ji}, \qquad J_3^F=\frac{1}{3}F_{ij}F_{jk}F_{ki} \tag{2}$$

$$K_1=K=\sigma'_{ij}F_{ji},\ K_2=\sigma'_{ij}F_{jk}F_{ki},\ K_3=\sigma'_{ik}\sigma'_{kj}F_{ji},\ K_4=\sigma'_{ik}\sigma'_{kj}F_{jl}F_{li} \tag{3}$$

where $\sigma'_{ij}=\sigma_{ij}-p'\delta_{ij}$ is the deviatoric stress tensor.

3 Results and Discussion

A total of over 80 numerical samples with different initial states (e.g., in terms of density and initial confining pressure) have been sheared to critical state under various monotonic loading paths. All samples reach a relatively steady state with constant stress and constant volume after an axial strain level at around 40% which satisfies the description of classic critical state. The soil state beyond this point has been taken as critical state. While certain degree of fluctuations in the soil response stays at this strain level, averages have been taken for the concerned quantities over a sustained stage of deformation, e.g., for an axial strain ranged between 40%~50%.

Data points for the critical void ratio are correlated to the mean effective stress p' in Fig. 1a. All data points collapse to a unique linear relation of the following general expression

$$e_c = e_\Gamma - \lambda_e \left(p'/p_a \right)^\xi \tag{4}$$

where $p_a = 101\ \text{kPa}$ is the atmospheric pressure. Similar linear relations have previously been found valid for Erksak sand and Toyoura sand. The second invariant of critical fabric anisotropy, $F_c = \sqrt{3F_{ij}^c F_{ij}^c / 2}$, is used to represent the degree of fabric anisotropy. Plotted in Fig. 1b is the correlation between F_c and p' at critical state. For each individual case of *b*, we observe a dependence of F_c on the critical mean effective stress according to the following power law:

$$F_c = m_F p'^\zeta \tag{5}$$

where m_F denotes a parameter dependent on the Lode angle or *b*, e.g., $m_F = \hat{m}_F(b)$. $\zeta = -0.14 \sim -0.09$. It is readily seen that a unique critical fabric structure independent of loading path is not attainable, according to the current definition of fabric tensor.

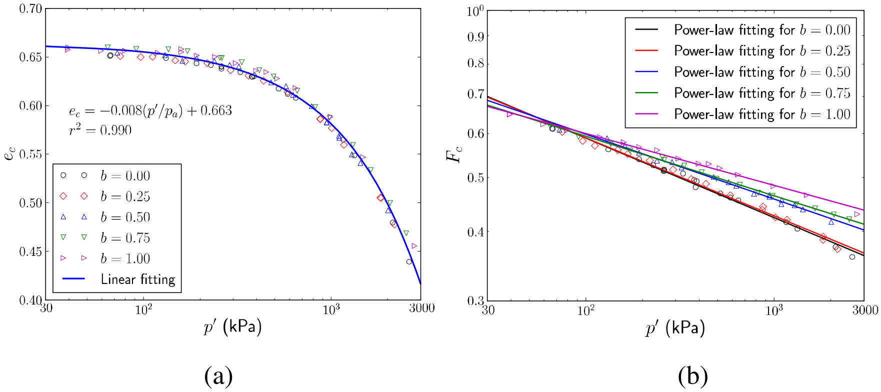
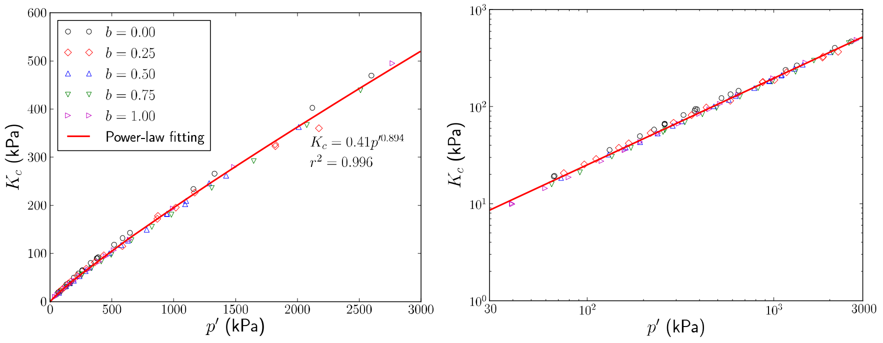


Fig. 1. Correlation between the critical void ratio (a) and the critical fabric anisotropy (b) with the mean effective stress

Based on contact-based fabric tensor, no unique property appears to be identifiable for the critical fabric anisotropy alone. However, the first joint invariant, K, has been found to correlate with p' at critical state in a unique way, as shown in Fig. 2. A striking power law correlation between the two is found to fit all data well:



(a) (b)

Fig. 2. Correlation between the first joint invariant K and p' at critical state by a power law fitting: (a) in a natural scale; (b) in a log-log plot

$$K_c = \alpha p'^{\varsigma} \tag{6}$$

where $\alpha = 0.41$ and $\varsigma = 0.894$ for the obtained data. A similar correlation exists for the fourth joint invariant, but no unique correlations can be found for the other two joint invariant. Based on Figs. 1 and 2 as well Eqs. (4) and (6), we see the three quantities e_c, K_c and p' are actually inter-correlated with one another. A spatial critical curve can then be plotted in the space of $e_c - p' - K_c$, as shown in Fig. 3. The projects of this spatial curve onto the plane of $e_c - p'$ and $K_c - p'$ givens the unique relation in Figs. 1 and 2 and Eqs. (4) and (6), respectively. This new critical state curve nicely unifies the classic critical state concept with the case in consideration of fabric anisotropy. While a contact-based fabric tensor has been used for the study, how to reconcile the finding with the Anisotropic Critical State Theory in [4] which was derived from a ***void-based*** fabric tensor is still under investigation.

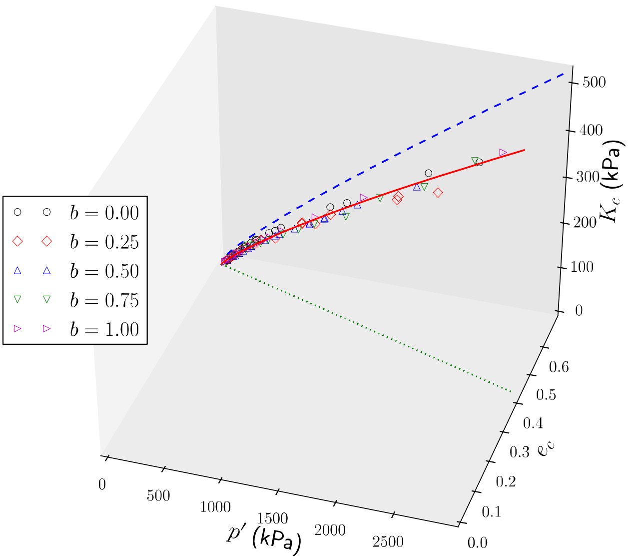


Fig. 3. A unique critical state line (CSL) in the space of $e_c - p' - K_c$

4 Conclusion

A unique relation between the first joint invariant of a *contact-based* fabric tensor and the deviatoric stress tensor has been identified at critical state of granular sand. Based on the relation, a unique critical state line (CSL) has been found in the space of $e_c - p' - K_c$ whose projection in the $e_c - p'$ plane is the CSL in classic critical state theory. The new CSL provides a practical unique definition of critical state in granular media in consideration of fabric anisotropy.

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