AIP | Conference Proceedings

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Citation: AIP Conf. Proc. **1542**, 229 (2013); doi: 10.1063/1.4811909
View online: http://dx.doi.org/10.1063/1.4811909
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1542&Issue=1
Published by the AIP Publishing LLC.

A New Definition on Critical State of Granular Media Accounting for Fabric Anisotropy

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Abstract. Conventional critical state concept for granular media lacks a proper reference to the anisotropic fabric structure developed at critical state, and is thus incomplete. This paper presents a micromechanical study to identify the characteristics of fabric anisotropy when a granular assembly reaches critical state. Our study reveals a strikingly unique, path-independent relationship between the mean effective stress and a fabric anisotropy parameter, K, defined by the first joint invariant of the deviatoric stress tensor and the deviatoric fabric tensor, at critical state. Data from over 80 DEM simulations under different loading conditions and intermediate stress ratios suggest a power law for this relationship. The new finding on critical fabric anisotropy is further incorporated into the conventional critical state conditions to redefine the critical state concept for granular media. The new definition dictates that the critical state corresponds to a unique state with constant stress, constant void ratio and constant K. It defines a unique spatial critical state line for a granular medium in the three-dimensional space K-e-p'. The projection of this spatial line onto the e-p' plane is the conventional (unique) critical state line.

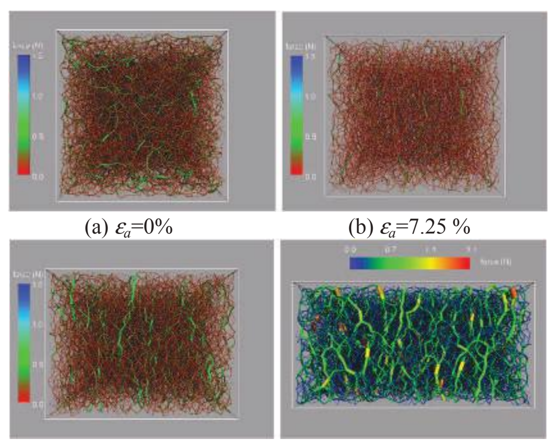
Keywords: Granular media, critical state, fabric anisotropy, soil mechanics, DEM.
PACS: 45.70.Vn; 61.43.-j; 83.50.Ax; 83.80.Fg.

INTRODUCTION

One of the most important findings in modern history of soil mechanics is the concept of critical state and the classic critical state theory by Roscoe and co-workers[1]. The classic definition of critical state in granular soils refers to a state of continuous shear deformation with a constant volume under constant stress, or equivalently, it corresponds to the following critical state conditions

$$\eta = \left(\frac{q}{p'}\right)_c = M_c,\ e = e_c = \hat{e}_c\left(p'\right). \qquad (1)$$

where q = the deviatoic stress; p' = the mean effective stress; M_c = the critical stress ratio; e_c = the critical void ratio. Evidently, the classical definition of critical state emphasizes *fabric isotropy* (the void ratio), but lacks a proper reference to *fabric anisotropy*. While both recent experimental and numerical studies indicated that the behavior of a granular soil under sustained shear is predominantly anisotropic, it remains so when the critical state is reached [2-5]. Shown indeed in Figure 1 depicts is the evolution of fabric structure, in terms of contact force network by DEM simulations, within a medium dense granular assembly subjected to undrained shear up to the critical state[5]. The anisotropic feature of the critical fabric structure is distinctly observable. Apparently there is a missing link preventing the classic critical state theory offering a comprehensive description of the critical state characteristics in granular media, as was recognized in the Anisotropic Critical State Theory (ACST) recently developed by Li & Dafalias[6].



(a) ε_a=0% (b) ε_a=7.25 % (c) ε_a=16.75% (d) ε_a=43.75 %

FIGURE 1. The evolution of fabric structure in a medium dense granular assembly subjected to undrained shear from an initial isotropic state (a) to the critical state (d), obtained by DEM simulations. The columns in the figure denote the contact normal force chain developed at particle contacts. The thickness of each column represents its magnitude.

The ACST takes into account the fabric anisotropy towards a more complete, elegant description of critical state. However, some postulates

Powders and Grains 2013
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in the ACST have been based on limited observations on 2D DEM simulations, which need to be rigorously corroborated by further experimental and numerical data. Serving initially to this purpose, the present study reaches some interesting new findings pertaining to the critical fabric anisotropy. Based on these findings, a new definition on critical state is proposed which is useful for future constitutive modeling of granular media.

METHEODOLOGY AND APPROACH

A 3D Discrete Element Method has been employed for the present study. In this DEM code we use spherical particles to generate granular assemblies and adopt a linear force-displacement contact law in conjunction with Coulomb's friction law to describe the inter-particle contacts[5,7-8]. The normal stiffness and tangential stiffness are set as $k_n/r = k_s/r = 100$ MPa, where r denotes the equivalent radius of two contacted particles. The inter-particle friction coefficient in Coulomb's law adopts the value $\mu = 0.5$, which is typical for quartz sand. By following a grain size distribution approximate to that of Toyoura sand, in a cubic container we randomly generate around 32,000 spherical particles with radii R ranging from 0.2 mm to 0.6 mm. A special technique used in Guo & Zhao[5] has been followed to produce isotropic granular assemblies with different initial void ratios. These different samples are then sheared to critical state following a number of different loading conditions, including both drained and undrained condition, constant p' tests and constant b [$b = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$, the intermediate principal stress ratio] commonly followed in laboratory tests. Examined in this study are a total of over 80 samples. For most cases the critical state is reached at an axial strain level around 45%~50%. No apparent shear banding localization has been observed in our simulations.

We employ the following definitions on the macroscopic quantities of interest derived from the discrete data of DEM results. The stress tensor proposed by Christoffersen *et al.*[9] is used

$$\sigma_{ij} = \frac{1}{V}\sum_{c\in N_c} f_i^c d_j^c, \tag{2}$$

where V = the total volume of the assembly; N_c = the total number of contacts; $\mathbf{f}^c$ = the contact force at a contact and $\mathbf{d}^c$ = the branch vector joining the centers of two contacted particles. Based on Eq. (2), one has $p' = \sigma_{ii}/3$, $q = \sqrt{3s_{ii}s_{ij}/2}$, where $s_{ij} = \sigma_{ij} - \delta_{ij}p'$ denoting the deviatoric stress (δ_{ij} = the Kronecker delta). To quantify the fabric anisotropy, we employ the following contact-normal based definition of fabric tensor proposed by Satake[10]

$$F_{ij} = \frac{15}{2}\left(\phi_{ij} - \frac{1}{3}\delta_{ij}\right),\quad \phi_{ij} = \frac{1}{N_c}\sum_{c\in N_c} n_i^c n_j^c. \tag{3}$$

where $\mathbf{n}^c$ = the unit contact normal vector at the contact plane of two contacted particles. The deviatoric fabric tensor F_{ij} is used in the subsequent discussion. With its first invariant is zero, its second and third invariants are of interest in the analysis

$$J_2^F = \frac{1}{2}F_{ij}F_{ji},\ J_3^F = \frac{1}{3}F_{ij}F_{jk}F_{ki}. \tag{4}$$

Meanwhile, the following joint invariants s_{ij} of F_{ij} and will also been used in the following discussion

$$K_1 = s_{ij}F_{ji},\ K_2 = s_{ij}F_{jk}F_{ki},\ K_3 = s_{ik}s_{kj}F_{ji},$$
$$K_4 = s_{ik}s_{kj}F_{jl}F_{li}. \tag{5}$$

In what follows, we use K to denote the first joint invariant K_1 for convenience.

RESULTS AND DISCUSSION

From the discrete data the critical-state quantities are retrieved and summarized. Figure 2 presents the critical stress ratio and critical void ratio (p_a is the atmospheric pressure). A unique critical stress ratio is found for each individual b case from the data points, while the critical void ratio is found bearing a unique linear relationship with the mean effective stress. These observations from Figure 2 appear to be fairly consistent with the classic critical state concept.

We further examine the property of critical fabric anisotropy F_{ij}. Figure 3 presents a correlation of the degree of anisotropy, F_c ($F_c = \sqrt{3J_2^F}$), with p', wherein a power-law function of the following form fits the data well for each case of b,

$$F_c = m_F\left(\frac{p'}{p_a}\right)^{\zeta}, \tag{6}$$

where m_F denotes a parameter dependent on the Lode angle (or b). Our fitting results indicate the power exponent $\zeta = -0.14$~-0.09 for b between 0 and 1. The b-dependence of F_c seems to be consistent with that for the critical stress ratio. It nevertheless lacks the unique property (of b-independence) observed on the critical void ratio. We have meanwhile found the third invariant of the fabric tensor is not unique either. To attain a unique characterization of the critical fabric anisotropy which is path-independent, we have sought for other quantities. Amongst many are the joint invariants defined in Eq. (5). Presented in Figure 4 is a correlation of the first joint invariant K with p' at critical state.

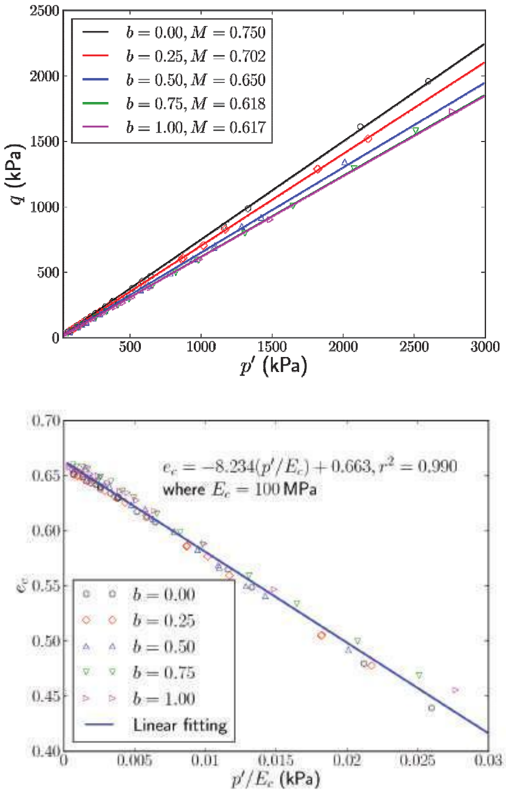


FIGURE 2. DEM data for the critical stress ratio (upper) and the critical state void ratio (bottom) and fittings.

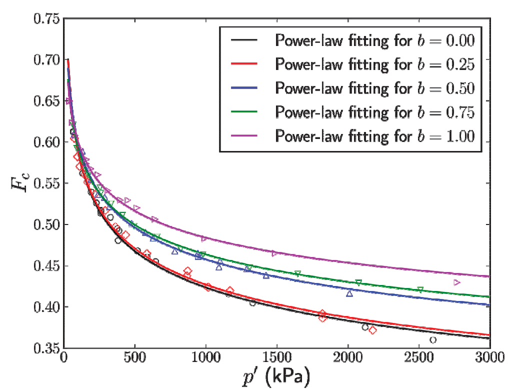


FIGURE 3. Correlation between the critical fabric anisotropy F_c (defined by the second invariant of F_{ij}) with the mean effective stress p'.

Figure 4 indicates that all data points of K_c~p' collapses to a unique curve which can be better described by a power law of the following form:

$$K_c = \alpha p'^{\xi}. \qquad (7)$$

where from the current study $\alpha = 0.41$ and $\xi = 0.89$. A linear fitting, also shown in Figure 4, is considered worse than the power law since it underestimates the data in the low pressure range and offers overestimations in the high pressure range. We also notice that K_4 defined in Eq. (5) shows a similar property to K_1, while the properties of K_2 and K_4 are non-unique at all (see Zhao and Guo[11-12] for detail).

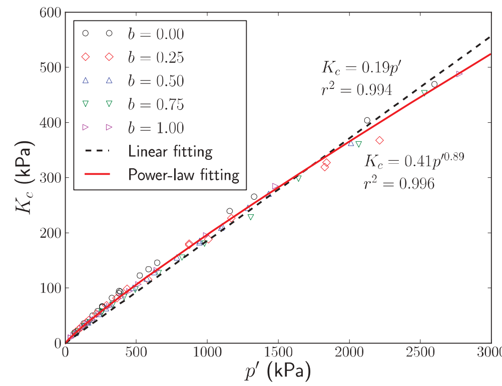
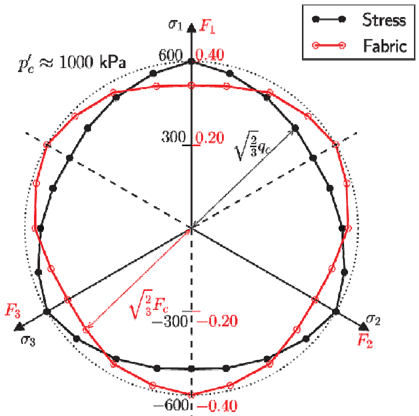


FIGURE 4. Correlation between the critical state values of the first joint invariant K with the mean effective stress p'.

It is interesting to further compare the state envelopes of the critical stress and the critical fabric anisotropy in the deviatoric stress. The upper figure in Figure 5 shows the critical stress surface and the critical fabric surface in the deviatoric plane obtained by different constant b tests at a pressure level of 1000 kPa. The critical state stresses form a smooth triangular-shaped surface which has a similar shape of Lade's failure criterion surface, while the critical fabric anisotropies form an envelope with an inverted smoothed triangular shape, reciprocal to that of the critical state stresses (c.f., Thornton and Zhang[13]).

While neither of the two envelopes is circular in the deviatoric plane (which render them path-independent), the complementary nature of their shapes suggests that a proper combination of the two may lead to a circular state envelope. The definition in Eq. (5) on the first (or the fourth) joint invariant offers exactly such an option. A further plot of K_c in the bottom of Figure 5 indeed confirms this expectation. As is shown, K_c does depict a perfect circular shape in the deviatoric plane which is independent of the Lode angle. This feature of path-independence renders it suitable for use as a reference state for soil state characterization.

In finding the unique dependence of both e_c and K_c on p', it is readily seen that the three are indeed inter-correlated in a unique manner. In the K-e-p' space this correlation proves to be a unique spatial critical state line which is shown in Figure 6. Orthogonal projection



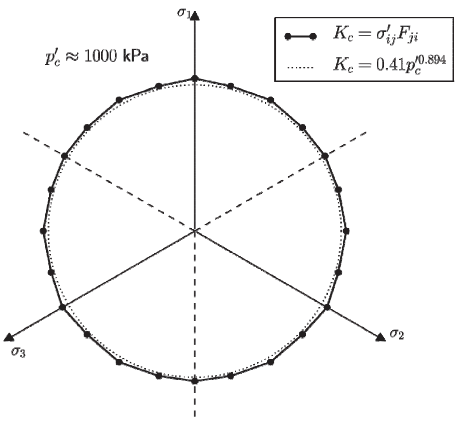


FIGURE 5. State surfaces of critical state stress and critical fabric anisotropy in the deviatoric plane (upper) and K_c in the deviatoric plane at p' = 1000 kPa.

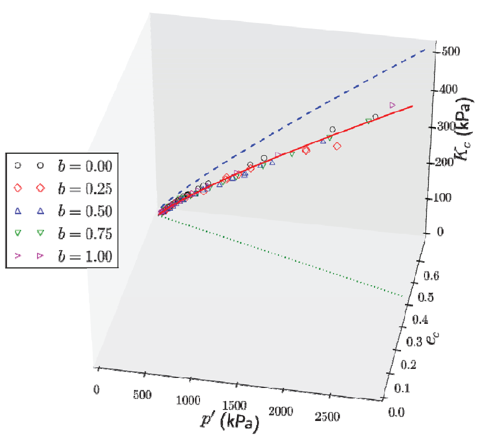


FIGURE 6. A unique critical state curve plotted in the space of *K-e-p'*.

of this spatial curve onto the *e-p'* plane turns out to the conventional critical state line used by classic critical state theory, while its orthogonal projection onto the *K- p'* plane is what Figure 4 shows. Since this spatial critical line is unique, both the projected critical state lines in the two planes are unique.

Based on the above findings, a new definition on critical state is proposed for granular media (*c.f.*, Li & Dafalias[5])

$$\eta = \left(q/p'\right)_c = M_c\left(b\right),\ e = e_c = \hat{e}_c\left(p'\right),$$
$$K = K_c = \hat{K}_c\left(p'\right). \tag{8}$$

ACKNOWLEDGMENT

The study was financially supported by Research Grants Council of Hong Kong (RGC/GRF 622910).

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