

Constitutive characterization of strength and deformation for natural clay and cemented sand

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ABSTRACT: Due to the influence of bonding and soil fabric, the behavior of a natural soil differs significantly from that of a remolded soil. It has been found that a soil with bonding may exhibit enhanced soil strength, stiffness as well as brittleness, and the soil fabric may lead to anisotropic soil behavior. A general failure criterion and constitutive framework considering both aspects of natural soils is of significant importance for practical analysis and design of geo-structures. In this paper, we first employ an anisotropic failure criterion to characterize the peak strength of cemented sands and natural clays. By replacing the constant frictional parameter with a hardening parameter, the anisotropic failure criterion is then modified to be a yield function, based on which a constitutive model is proposed to describe the effect of bonding and fabric anisotropy on the behavior of naturally bonded soil. Simulations of the behavior of cemented sands by the model agree favorably with test results.

1 INTRODUCTION

Natural soils are routinely dealt with in geotechnical engineering. It has long been observed that the behavior of natural soils differs significantly from that of reconstituted soils. This is due to intrinsic structure in natural soils existing in form of bonding and fabric (Burland, 1990). The bonding of natural soils normally originates from the sedimentation process (Mitchell & Soga, 2005). Meanwhile, cementation has often been used in soil improvement to enhance the soil strength, stiffness as well as the resistance to liquefaction (e.g. Ismael, 1999; Porbaha et al., 2000; Gallagher & Mitchell, 2002), during which bonding may be formed. Fabric anisotropy of natural soils is formed during the deposition and compaction processes and usually takes the form of cross-anisotropy (or transverse-isotropy) characterized by one direction with distinctive anisotropy perpendicular to a bedding or lamination plane wherein it is largely isotropic (e.g. Casagrande & Carillo, 1944; Callisto & Calabresi, 1998; Miura & Toki, 1984; Lade & Kirkgard, 2000). This perpendicular direction, normally coincident with the direction of deposition, is referred to as the axis of anisotropy.

Bonding and fabric anisotropy play different roles in affecting the behavior of natural soils. Soil bonding may lead to increased peak strength, stiffness as well as brittleness of soils (e.g., Clough et al. 1981; Horpibulsuk, et al., 2005; Wang & Leung, 2008). It is also found that cemented soils are prone to dilate, which leads to higher liquefaction resistance (e.g. Saxena et al., 1988; Gallagher & Mitchell, 2002). The importance of fabric anisotropy to soil behavior has long been recognized in

both sand (e.g. Oda et al., 1978; Miura & Toki, 1984) and clay (e.g. Callisto & Calabresi, 1998; Lade & Kirkgard, 2000). For example, it has been observed that soil strength and stiffness are higher in conventional triaxial compression and lower in conventional triaxial extension (e.g. Miura & Toki, 1984; Callisto & Calabresi, 1998). It is also found that, as the major principal stress direction deviates from the direction of deposition, the soil becomes more contractive, and the resistance to liquefaction decreases accordingly (e.g. Kumruzzaman & Yin, 2010; Yoshimine et al., 1998; Miura & Toki, 1984).

Evidently, soil structure may have an important impact on the strength and deformation characteristics of natural soil, and should be carefully considered in the analysis and design of geotechnical structures. In particular, to accurately characterize the strength of natural soils, a general failure criterion and suitable constitutive framework are indeed needed. While both isotropic and anisotropic failure criteria have been proposed to characterize the failure of geomaterials (see a detailed review by Gao et al., 2010), limited attempts have been made towards modeling the failure and deformation characteristics of natural soils. In this paper, we will first apply a newly developed anisotropic failure criterion (Gao et al., 2010) which has been verified by test data on a wide range of geomaterials, such as sand, clay and rock, to characterize the failure of natural soils. Special attention will be paid on the behavior of cemented sand and natural clay. Regarding the constitutive modeling of natural soils, existing studies have been devoted to either of the two aspects (bonding and fabric anisotropy) of natural clays (e.g. Rouainia, & Muir Wood, 2000; Asaoka et al., 2000; Liu & Carter,

2002), but rarely two of them simultaneously for sand (e.g. Hirai et al., 1989; Sun & Matsuoka, 1999; Hicher et al., 2008; Li & Dafalias, 2002). Indeed, Michalowski (2008) has found that the orientation of fibres in artificially cemented soils is usually anisotropic and may cause strongly anisotropic behavior in the soil. It is thereby desirable to develop a model that can take into account of the bonding and fabric anisotropic effect in a comprehensive way to treat cases like this. This shall be attempted in this paper. A simple elasto-plastic will be developed here to investigate the behavior of sandy soils by considering the effect of bonding, de-bonding process as well as inherent fabric anisotropy. The yield function is adapted from an anisotropic failure criterion previously proposed by the authors (Gao et al., 2010). Test results on cemented Ottawa sand (Wang & Leung, 2008) and Multiple-sieving-pluviated (MSP) Toyoura sand (Miura & Toki, 1984) will be used to verify the model performance.

2 A GENERAL ANISOTROPIC FAILURE CRITERION FOR GEOMATERIALS

The anisotropic failure criterion proposed by Gao et al. (2010) is in the following form

$$\alpha\sqrt{\bar{I}_1^2 - 3\bar{I}_2} + \frac{2(1-\alpha)\bar{I}_1}{3\sqrt{(\bar{I}_1\bar{I}_2 - \bar{I}_3)/(\bar{I}_1\bar{I}_2 - 9\bar{I}_3)} - 1} = \frac{1}{3}M_f f(A)\bar{I}_1 \quad (1)$$

where M_f is a frictional parameter; α is a model parameter ; $\bar{I}_1 = (\bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3)$, $\bar{I}_2 = (\bar{\sigma}_1\bar{\sigma}_2 + \bar{\sigma}_2\bar{\sigma}_3 + \bar{\sigma}_1\bar{\sigma}_3)$ and $\bar{I}_3 = (\bar{\sigma}_1\bar{\sigma}_2\bar{\sigma}_3)$ are the three invariants of a transformed stress tensor $\bar{\sigma}_{ij}$ defined below

$$\bar{\sigma}_{ij} = \sigma_{ij} + \left[p_r \left(\frac{p + \sigma_0}{p_r} \right)^n - p \right] \delta_{ij} \quad (2)$$

where δ_{ij} is the Kronecker delta; σ_{ij} is the commonly referred Cauchy stress tensor; p_r is a reference pressure; σ_0 is the triaxial tensile strength of a material; n is a model parameter. Key to the anisotropic failure criterion is the addition of the function $f(A)$ defined below to introduced the influence of anisotropy

$$f(A) = \exp\left\{d\left[(A+1)^2 + \beta(A+1)\right]\right\} \quad (3)$$

where d and β are material constants. A is an anisotropic variable reflective of the influence of loading direction with respect to fabric, defined as follows

$$A = \frac{s_{ij}d_{ij}}{\sqrt{s_{mn}s_{mn}}\sqrt{d_{pq}d_{pq}}} \quad (4)$$

where $s_{ij} = \sigma_{ij} - \sigma_{kk}\delta_{ij}/3$, $d_{ij} = F_{ij} - F_{kk}\delta_{ij}/3$. F_{ij} is the fabric tensor defined by Oda & Nakayama (1989). For a cross-anisotropic material, assuming that the principal axes of fabric align in the reference coordinate (x_1, x_2, x_3) with the $x_2 - x_3$ plane being the isotropic plane, F_{ij} can be expressed as,

$$F_{ij} = \begin{bmatrix} F_1 & 0 & 0 \\ 0 & F_2 & 0 \\ 0 & 0 & F_3 \end{bmatrix} = \frac{1}{3+\Delta} \begin{bmatrix} 1-\Delta & 0 & 0 \\ 0 & 1+\Delta & 0 \\ 0 & 0 & 1+\Delta \end{bmatrix} \quad (5)$$

where Δ is a scalar that characterizes the magnitude of the cross-anisotropy. Its value ranges from zero when the material is absolutely isotropic, to unity when the degree of anisotropy is the maximum. Note that when $f(A) \equiv 1$, the anisotropic failure criterion becomes identical to the underlying isotropic failure criterion as in Yao et al. (2004). The primary effect of the function $f(A)$ is to change the shape of the underlying isotropic failure surface in the deviatoric plane. When $f(A) > 1$, it plays a role to expand the failure surface with respect to the isotropic one, and to shrink it when $f(A) < 1$.

3 APPLICATION OF THE FAILURE CRITERION TO NATURAL SOILS

In Gao et al. (2010), the failure criterion has been verified by a comparison of the model predictions with test results on sand, clay and rock. In the following sub-sections, we shall further apply it to cemented sands and natural clays.

3.1 Cemented sand

Reddy & Sexena (1993) performed a series of true triaxial tests under constant mean stress on Monterey No. 0 sand with 2% of Portland cement type I. Complementary conventional triaxial tests were also conducted to investigate the failure characteristics in the meridian plane. Shown in Fig. 1 is the comparison between the test data and the prediction of the failure criterion presented in Eq.(1) in both the meridian plane and the deviatoric plane. Since the anisotropic effect has not been investi-

gated in the tests, the isotropic criterion is employed to perform the simulations. Note that the parameter M_f is determined by best fitting all available test data in triaxial compression, with each data point in the deviatoric plane corresponding to a single test. It is noticed that the criterion slightly underestimates the test results in the deviatoric plane, especially at $b=0.75$. Nevertheless, the coincidence between the test results and criterion simulations is satisfactory. Note that the distances of the data points from the origin in the deviatoric plane are set to $\sqrt{2/3}q$ throughout the paper, where q is the deviatoric stress.

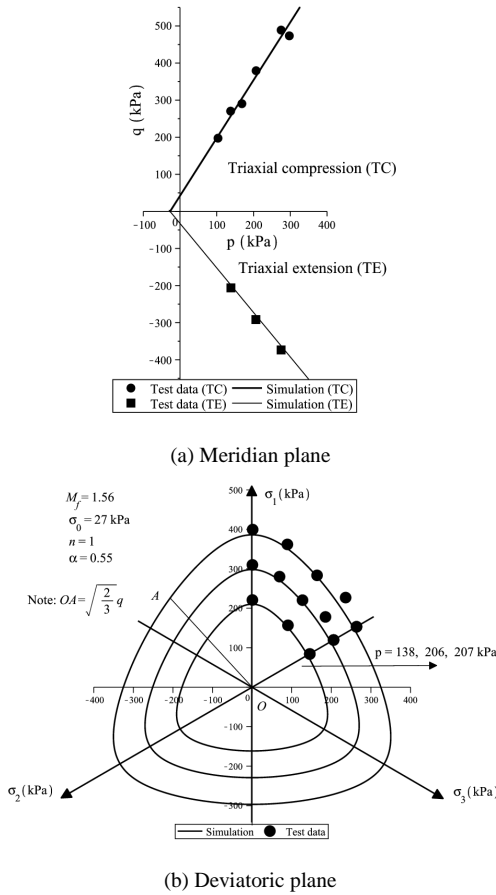


Fig. 1 Comparison between the test results on cemented Monterey No. 0 sand and simulations of the failure criterion in Eq.(1)

3.2 Natural clay

Stress path-controlled triaxial and true triaxial tests have been carried out by Callisto & Calabresi (1998) on a natural soft clay, Pisa clay. The soil tested was sampled from the upper clayey deposit found below the Tower of Pisa. All the samples were reconsolidated to the in-situ stress state and then sheared to failure with a constant mean stress of 88.2 kPa under drained conditions. Since there is no sufficient data to determine all param-

eters required for characterizing the failure in the meridian plane (M_f , σ_0 and n), only M_f is determined according to the stress at failure at $\theta=0$ (see Gao et al. (2010) for the definition) by setting $\sigma_0=0$ and $n=1$. The rest parameters α , d and β are determined according to the procedure discussed in Gao et al. (2010). As can be seen from the comparison in Fig. 2, the isotropic criterion overestimates the strength in the range of $120^\circ \leq \theta \leq 180^\circ$, while the anisotropic criterion captures the overall trend satisfactorily. Note that the value of α in both the isotropic and anisotropic failure criteria is the same.

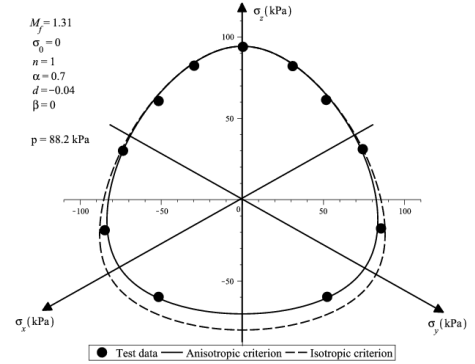


Fig. 2 Test results on the failure of natural Pisa clay and model predictions by both the isotropic and anisotropic failure criteria

4 A SIMPLE ELASTO-PLASTIC CONSTITUTIVE MODEL FOR SAND SOILS

In this section, the anisotropic failure criterion will be extended to model the deformation and yielding of cemented sandy soils. In particular, the bonding, de-bonding and the inherent fabric anisotropic effect will be taken into account.

4.1 Elastic moduli

The following nonlinear relations are employed to describe the elastic shear modulus G and bulk modulus K in sand

$$G = G_0 \exp\left(\frac{\sqrt{\sigma_0}}{\sqrt{p_r}}\right) \frac{(2.97 - e)^2}{1 + e} p_r \sqrt{\frac{p}{p_r}} \quad (6)$$

$$K = G \frac{2(1 + \nu)}{3(1 - 2\nu)} \quad (7)$$

where G_0 is a material constant; e is the void ratio; ν is the Poisson's ratio. The additional term $\exp\left(\sqrt{\sigma_0/p_r}\right)$ is introduced to improve the

model description of the elastic behavior in cemented soils.

4.2 Yield function

We extend the anisotropic failure criterion in Eq.(1) to a yield function, by replacing M_f with a hardening parameter H as follows, and use it in the proposed model

$$f = \alpha\sqrt{\bar{I}_1^2 - 3\bar{I}_2} + \frac{2(1-\alpha)\bar{I}_1}{3\sqrt{(\bar{I}_1\bar{I}_2 - \bar{I}_3)/(\bar{I}_1\bar{I}_2 - 9\bar{I}_3)} - 1} - \frac{1}{3}Hg(A)\bar{I}_1 = 0 \quad (8)$$

4.3 Hardening law

In line with the yield function in Eq.(8), the following hardening law is employ for H ,

$$dH = \langle dL \rangle r_H = \langle dL \rangle \frac{Gc_h\zeta}{Hp_r} (M_f - H) \quad (9)$$

where dL is a loading index and $\langle \cdot \rangle$ denotes the Macauley bracket with $\langle x \rangle = 0$ when $x \leq 0$ and $\langle x \rangle = x$ when $x > 0$. c_h is a positive model parameter. ζ is a scaling factor to take into account the effect of fabric anisotropy in the soil stiffness,

$$\zeta = \exp[-k(A+1)] \quad (10)$$

where k is a positive model parameter, which renders ζ a decreasing function of A . This is consistent with experimental observations that, under otherwise identical conditions, the soil response becomes softer as the major principal stress direction deviates away from the direction of deposition (e.g. Yoshimine et al., 1998; Miura and Toki, 1984). Note that $\zeta \equiv 1$ in conventional triaxial compression ($A = -1$), which renders this shear model a convenient reference for model calibration.

Experimental observations (e.g., Clough et al. 1981; Schnaid et al., 2001) show that the bonding of soils is gradually damaged due to plastic deformation, which results in the degradation of shear modulus after peak strength. In this model, the amount of de-bonding is simply assumed to be proportional to the plastic deviatoric strain increment as follows,

$$d\sigma_0 = \langle dL \rangle r_0 \quad (11)$$

where r_0 denotes the evolution direction of σ_0 and is expressed as

$$r_0 = \begin{cases} -m \frac{p}{p_r} \sigma_0 & \text{for } \sigma_0 > 0 \\ 0 & \text{for } \sigma_0 = 0 \end{cases} \quad (12)$$

where m is a positive parameter. Such an evolution law renders that the bonding keeps decreasing with the plastic deformation process as long as $\sigma_0 > 0$. Once the bonding is totally damaged, it can not be recovered by pure plastic deformation as r_0 stays zero.

4.4 Dilatancy and flow rule

To include the effect of bonding and fabric anisotropy in the dilatancy, we propose the following dilatancy equation,

$$D = \frac{d\varepsilon_v^p}{\sqrt{\frac{2}{3}de_{ij}^pde_{ij}^p}} = \frac{d_1}{\exp\left(\int \langle dL \rangle\right)} (M_p d_C d_F - H) \quad (13)$$

where $d\varepsilon_v^p$ is the plastic volumetric strain increment, e_{ij}^p is the plastic deviatoric strain increment, d_1 is a positive model parameter, M_p is the phase transformation stress ratio measured in conventional undrained triaxial compression tests on remolded samples. The role of denominator is to control the volume change given by this dilatancy equation. As the sample is sheared to the critical state, the plastic deviatoric strain increment is infinite, which makes the value of D approach 0 (Li & Dafalias, 2004). The two scaling factors d_C and d_F are used to characterize the bonding and anisotropic effect respectively,

$$d_C = \exp\left(-c_0\sqrt{\frac{\sigma_0}{p_r}}\right) \quad (14)$$

$$d_F = \exp[k(A+1)] \quad (15)$$

where c_0 a positive model parameter and k is the same as that in the expression of ζ . It can be seen from the expression for d_C that, as the bonding effect increases, the phase transformation stress ratio decreases. In other words, the soil is more prone to dilate and the liquefaction resistance increases. This is in accordance with the experimental observations (e.g., Saxena et al., 1988; Gallagher & Mitchell, 2002). The term d_F renders the phase transformation ratio increases as the major principal stress direction deviates more from the direction of deposition, which implies the sand is more prone to liquefy (e.g., Yoshimine et al., 1998; Miura & Toki, 1984).

Furthermore, associated flow rule is assumed in this paper,

$$de_{ij}^p = \langle dL \rangle n_{ij} \quad (16)$$

where n_{ij} is a unit tensor defined as

$$n_{ij} = \frac{\frac{\partial f}{\partial \sigma_{ij}} - \frac{1}{3} \frac{\partial f}{\partial \sigma_{mn}} \delta_{mn} \delta_{ij}}{\left| \frac{\partial f}{\partial \sigma_{ij}} - \frac{1}{3} \frac{\partial f}{\partial \sigma_{mn}} \delta_{mn} \delta_{ij} \right|} \quad (17)$$

5 MODEL VERIFICATION

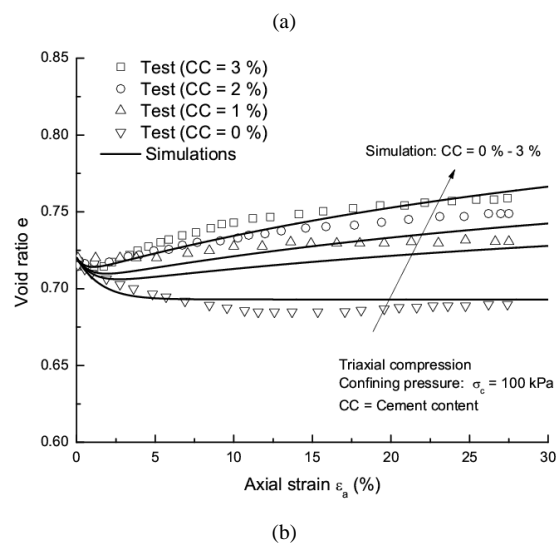
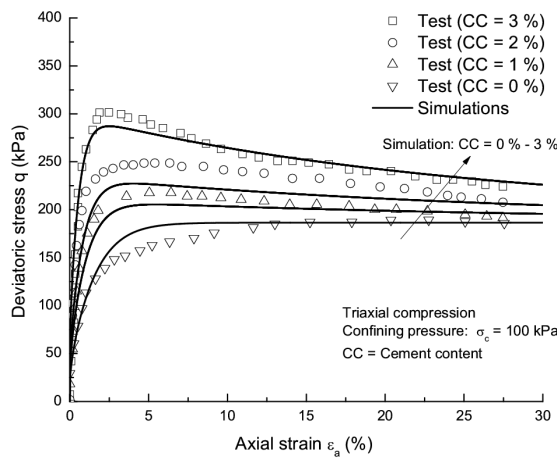


Fig. 3 Comparison between the model predictions and tested results on cemented Ottawa sand in triaxial compression

We demonstrate the predictive capability of the model in characterizing the structure effect on sandy soils in this section. Fig. 3 shows a comparison between the model simulation and tested stress-strain relations on cemented Ottawa sand (Wang & Leung, 2008). Evidently, the model cap-

tures the general trend satisfactorily. Nevertheless, it tends to under-estimate the peak strength of cemented samples, especially in the case the cement content is 2%. This is due to that the initial value of the triaxial tensile strength σ_{0i} has been determined by neglecting the de-bonding before the peak strength state. Better prediction can be achieved by specifying slightly greater value of σ_{0i} in this case. Meanwhile, from the comparison we see the model gives less volume contraction for the un-cemented samples and less volume expansion for the cemented samples. Fig. 4 shows a comparison between the model simulations and the test results on the multiple-sieving pluviated (MSP) Toyoura sand (Miura & Toki, 1984). Again, reasonable agreement is found between the two. The proposed model appears to be capable of providing reasonable predictions on the behavior of cemented sandy soils.

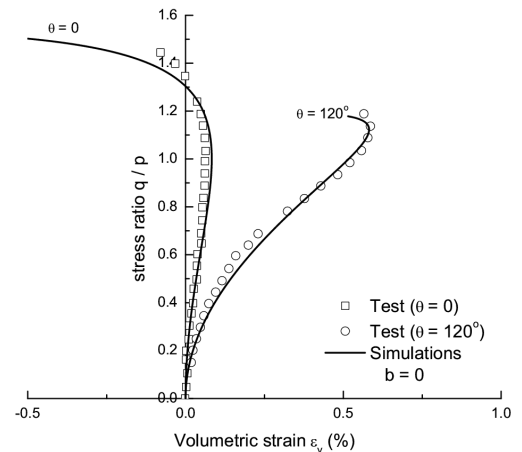
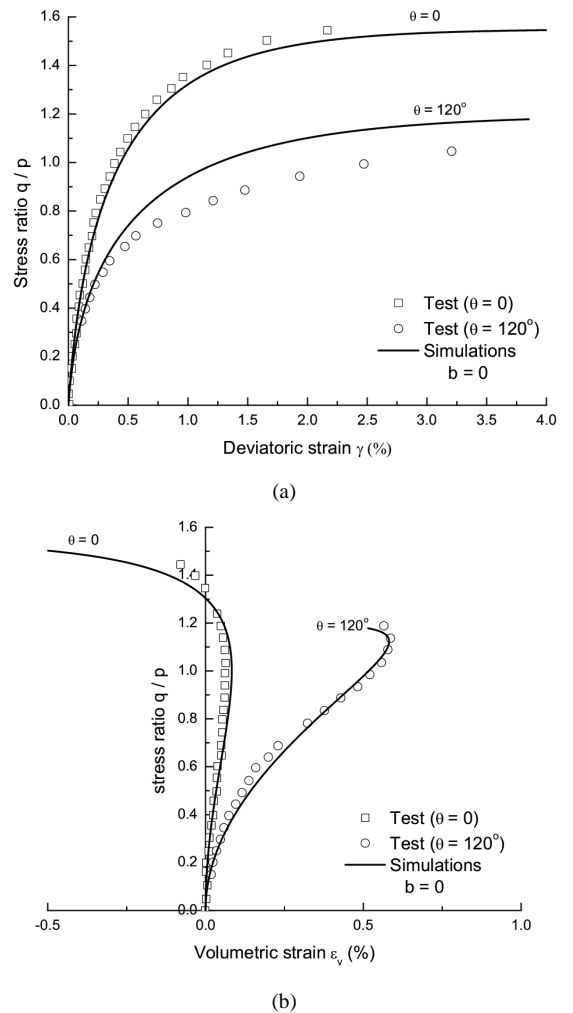


Fig. 4. Comparison between the model simulations and test results on MSP Toyoura sand in true triaxial tests.

6 CONCLUSION

In this paper, an anisotropic failure criterion is first applied to the prediction of strength for natural clays and cemented sands. Based on the fabric tensor proposed by Oda and Nakayama (1989), an anisotropic variable A is introduced to characterize the relative orientation between the soil fabric and loading direction. Comparison between the predictions and test results on cemented sand and natural clay demonstrates that the anisotropic failure criterion can address the structure effect on soil strength reasonably.

The anisotropic failure criterion has also been generalized to model the behavior of cemented sands, with bonding, de-bonding and inherent fabric anisotropy being carefully considered. In this model, the anisotropic failure criterion is extended to a yield function by replacing the constant frictional coefficient with a hardening parameter. The conventional triaxial compression on remolded samples is made as a reference for model calibration. All parameters can be optimized based on the conventional triaxial compression and extension test results. The model has been verified by test results on cemented Ottawa sand (Wang & Leung, 2008) and MSP Toyoura sand (Miura & Toki, 1984), and model predictions compare well with the test data.

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