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Constitutive modelling of fabric anisotropy in sand

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ABSTRACT: Fabric and its evolution have significant effect on the mechanical behaviour of granular materials. A three-dimensional anisotropic model for granular material is proposed with proper consideration of fabric evolution. An explicit expression for the yield function is proposed in terms of the invariants and joint invariants of the stress ratio and fabric tensors. The material fabric is assumed to evolve with plastic shear deformation in a manner that its principal axes tend to become co-directional with those of the loading direction and its magnitude approaches a critical state value at large deformation. A non-coaxial and associated flow rule in the deviatoric stress space is employed based on the yield function. The model is capable of characterizing the complex anisotropic behaviour of granular materials under monotonic loading with fixed principal stress directions and meanwhile gives reasonable explanation for the micromechanical mechanism for static liquefaction and noncoaxiality between the stress and plastic strain increment axes.

1 INTRODUCTION

Transversely anisotropic fabric structure is commonly observed in both natural and manmade sand deposits and profoundly influences the mechanical behaviour of these soils including strength and dilatancy (Yoshimine et al. 1998, Gao et al. 2010). Proper consideration of the effect of fabric is important for safe design and maintenance of relevant key infrastructures (Uthayakumar & Vaid 1998).

There have been many attempts on theoretical characterization and modelling of fabric anisotropy in sand and its effect on macroscopic sand behaviour. Among many, those models based on the use of rotated yield and plastic potential surfaces have gained limited popularity in the literature (Sekiguchi & Ohta 1977, Pestana & Whittle 1999). However, yield surface rotation may not be able to account for the anisotropic nature of sand related to particle orientation, contact normal and void space distribution properly, as the magnitude and direction of rotation is typically associated with the initial stress state (Kaliakin 2003). The employment of fabric tensors derived from the microstructural information of sand has proved to be efficient and physically more realistic in modelling sand behaviour (Oda & Nakayama 1989, Pietruszczak 1999, Li & Dafalias 2002, Dafalias et al. 2004). Being successful to a certain extent, these studies have commonly ignored the change of the fabric anisotropy during the deformation of the material, which is at odd with both experimental and numerical observations, as the sand fabric will adjust to sustain the external loading in an optimum manner when it is deformed (Li & Li 2009, Zhao & Guo 2013; Guo & Zhao, 2013).

This paper presents an anisotropic sand model accounting for fabric evolution based on the anisotropic critical state theory (ACST) by Li & Dafalias (2012). The new model features an explicit yield function expressed in terms of the invariants and joint invariants of the stress ratio tensor r_{ij} and a deviatoric fabric tensor F_{ij}. Over a typical monotonic loading course, the fabric tensor is assumed to evolve towards the direction of loading. Based on the proposed framework, a non-coaxial flow rule is readily derived.

2 CONSTITUTIVE MODEL

2.1 *Yield function*

Based on this micromechanical deformation mechanism that the shear resistance of sand is contributed by inter-particle friction and fabric anisotropy, we propose the following yield function,

$$f = \frac{R}{g(\theta)} - He^{-k_h(A-1)^2} = 0 \qquad (1)$$

where $R = (3r_{ij}r_{ij})^{0.5}$ with $r_{ij} = (\sigma_{ij} - p\delta_{ij})/3$ being the stress ratio tensor, in which σ_{ij} is the stress tensor, $p = \sigma_{ij}/3$ is the mean normal stress, δ_{ij} is the Kronecker delta; H is a hardening parameter whose evolution law depends on the stress as well as internal variables including soil density and fabric; A is a fabric anisotropy variable; k_h is a non-negative model constant with default value of 0.03; $g(\theta)$ is an interpolation function based on the Lode angle θ of r_{ij} as follows (personal communication, Z.L. Wang 1992)

$$g(\theta) = \frac{\sqrt{(1+c^2)^2 + 4c(1-c^2)\sin 3\theta} - (1+c^2)}{2(1-c)\sin 3\theta} \qquad (2)$$

where $c = M_e/M_c$ is the ratio between the critical state stress ratio in triaxial extension M_e and that in triaxial compression M_c.

An important inclusion in the yield function in Equation 1 is a fabric anisotropy variable A that is defined by the following joint invariant of F_{ij} and n_{ij} (Li & Dafalias 2004, Gao et al. 2014)

$$A = F_{ij}n_{ij} \qquad (3)$$

where F_{ij} is a symmetric traceless tensor whose norm $F = (F_{ij}F_{ij})^{0.5}$ is referred to as the degree of fabric anisotropy. For convenience, F_{ij} is normalized such that in critical state, F is unity. The deviatoric unit loading direction tensor n_{ij} in Equation 3 is defined as follows (Li & Dafalias 2004)

$$n_{ij} = \frac{N_{ij} - N_{mn}\delta_{mn}\delta_{ij}/3}{\left\| N_{ij} - N_{mn}\delta_{mn}\delta_{ij}/3 \right\|} \qquad (4)$$

with

$$N_{ij} = \frac{\partial[R/g(\theta)]}{\partial R}\frac{\partial R}{\partial r_{ij}} + \frac{\partial[R/g(\theta)]}{\partial g(\theta)}\frac{\partial g(\theta)}{\partial \theta}\frac{\partial \theta}{\partial r_{ij}} \qquad (5)$$

Obviously, $n_{ii} = 0$ and $n_{ij}n_{ij} = 1$. Notice that the n_{ij} is the deviatoric unit normal to a yield surface resulting from Equation 1 with the assumption that A is a constant (in other words n_{ij} is not normal to the yield surface of Equation 1).

2.2 *Evolution law for H and F_{ij}*

Within the hypothesis that sand's stress-strain response is incrementally linear, the evolution of the two internal variables is assumed to be

$$dH = \langle L\rangle \frac{G(1-c_h e)}{pR}\left[M_c g(\theta)e^{-n\zeta} - R\right] \qquad (6)$$

$$dF_{ij} = \langle L\rangle \Theta_{ij} = \langle L\rangle k_f (n_{ij} - F_{ij}) \qquad (7)$$

where c_h, n and k_f are three model parameters, e is the current void ratio, $\langle\ \rangle$ are the Macauley brackets such that $\langle L\rangle = L$ for $L > 0$ and $\langle L\rangle = 0$ for $L \le 0$, ζ is the dilatancy state parameter defined as below (Li & Dafalias 2012)

$$\zeta = \psi - e_A(A-1) \qquad (8)$$

where e_A is a model parameter, $\psi = e - e_c$ is the state parameter defined by Been & Jefferies (1985) with e_c being the critical state void ratio corresponding to the current p. In the present work, the critical state line in the e-p plane is given by (Li & Wang 1998)

$$e_c = e_r - \lambda_c (p/p_a)^{0.7} \qquad (9)$$

where e_r and λ_c are two material constants and p_a (=101 kPa) is the atmospheric pressure. The above evolution law of F_{ij} with plastic deformation expressed by Equation 7 leads towards coaxiality with the loading direction n_{ij}.

2.3 *Dilatancy relation and flow rule*

A proper dilatancy relation D defined as below is essential for modelling the sand behaviour

$$D = \frac{d\varepsilon_{ii}^p}{\sqrt{2de_{ij}^p de_{ij}^p/3}} \qquad (10)$$

where $d\varepsilon_{ij}^p$ is the plastic strain increment and de_{ij}^p is the plastic shear strain increment. Based on Li & Dafalias (2012), the following dilatancy relation which accounts for the effect of density, confining pressure and anisotropy is proposed

$$D = \frac{d_1\left[1 + R/M_c g(\theta)\right]}{M_c g(\theta)}\left[M_c g(\theta)e^{m\zeta} - R\right] \qquad (11)$$

where d_1 and m are two model constants.

By assuming an associated flow rule in the deviatoric stress space based on the yield function in Equation 1, the increment of the plastic shear strain de_{ij}^p is expressed as

$$de_{ij}^p = \langle L\rangle m_{ij} \qquad (12)$$

where

$$m_{ij} = \frac{\partial f/\partial r_{ij} - (\partial f/\partial r_{mn})\delta_{mn}\delta_{ij}/3}{\left\| \partial f/\partial r_{ij} - (\partial f/\partial r_{mn})\delta_{mn}\delta_{ij}/3 \right\|} \qquad (13)$$

Notice that m_{ij} is normal to the yield surface expressed by Equation 1. Since $\partial f/\partial r_{ij}$ consists of two parts with one being coaxial with r_{ij} (or equivalently σ_{ij} itself) and the other involving F_{ij} which is attributed to fabric anisotropy and is in general non-coaxial with r_{ij} (Gao et al. 2014), the flow rule expressed by Equations 12 and 13 naturally address the non-coaxiality issue in soil modelling.

Table 1. Model parameters for Toyoura sand.

Parameter	Value
G_0	125
ν	0.1
M_c	1.25
c	0.75
e_r	0.934
λ_c	0.02
c_h	0.90
n	3.0
d_1	0.2
m	5.3
e_A	0.10
k_f	5.7

2.4 *Elastic moduli*

As plastic strain dominates sand deformation, the influence of elastic anisotropy, if any, is considered negligible. The following elastic moduli (Richart et al. 1970, Li & Dafalias 2012, Gao et al. 2014) are employed:

$$G = G_0 \frac{(2.97 - e)^2}{1 + e} \sqrt{p p_a} \tag{14}$$

$$K = G \frac{2(1+\nu)}{3(1-2\nu)} \tag{15}$$

where G and K denote the elastic shear and bulk modulus, respectively, G_0 is a material constant and ν is the Poisson's ratio assumed to be a constant.

3 MODEL SIMULATION FOR ANISOTROPIC SAND BEHAVIOUR

3.1 *Model parameters*

To verify the model capability in simulating the anisotropic sand behaviour, we employ the test data for the dry-deposited Toyoura sand reported by Yoshimine et al. (1998). The model parameters are listed in Table 1 and the initial degree of anisotropy F_0 is set to be 0.45. The procedure for parameter determination is discussed in Gao et al. (2014).

3.2 *Model simulation*

Figure 1 shows the model simulations for the anisotropic sand behaviour under undrained torsional loading with constant intermediate principal stress vatiable $b = 0.25$. In this figure, α is the major principal stress direction with respect to the deposition direction and D_{rc} is the relative density after consolidation. Clearly, the model well captures the trend that larger value of α generally leads to softer (lower shear stress σ_1-σ_3 at the same deviatoric strain ε_1-ε_3) and relatively more contractive sand response.

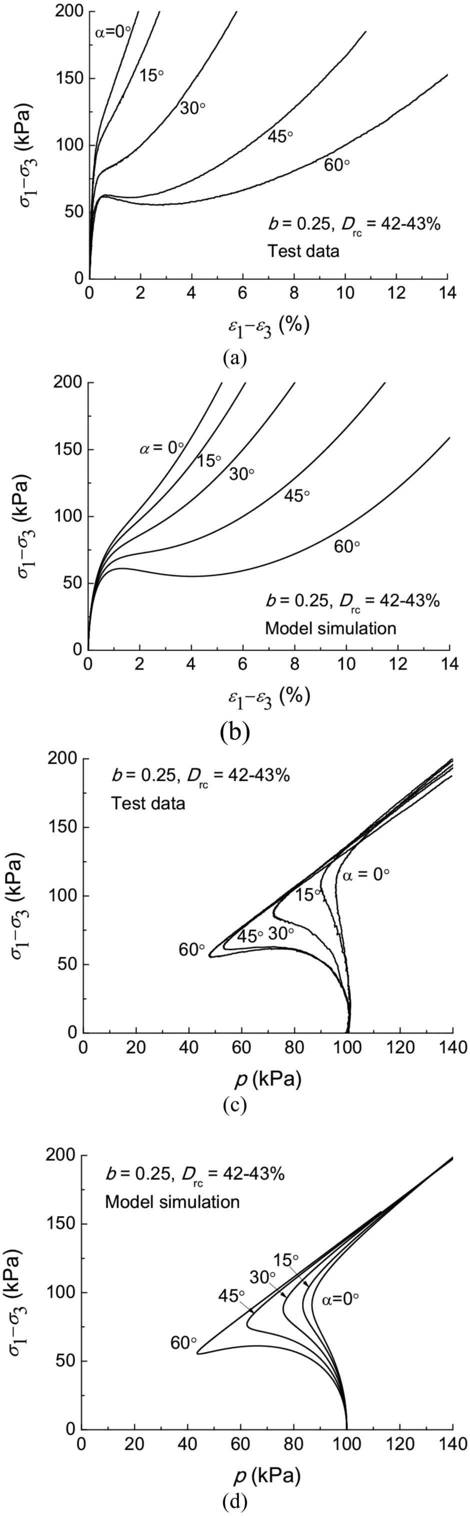

Figure 1. Model simulation for the anisotropic sand behaviour in undrained torsional shear tests (data from Yoshimine et al. 1998).

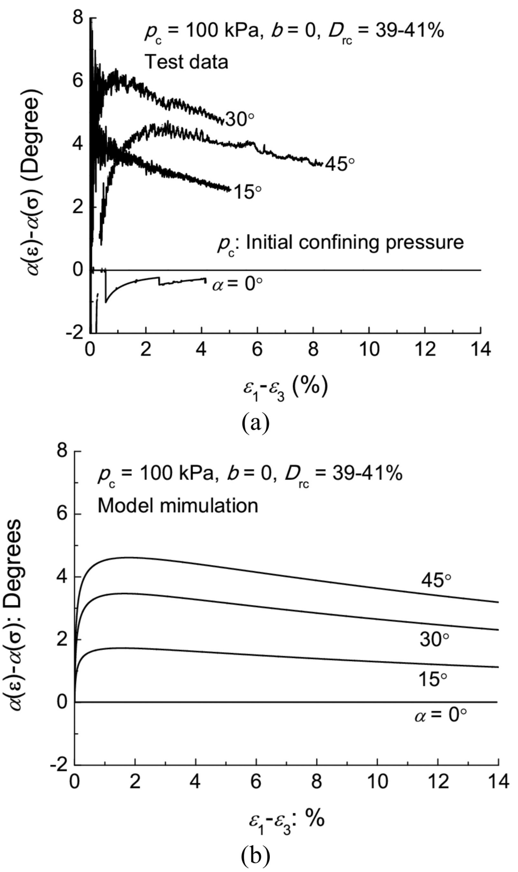

Figure 2. Comparison between the observed and model predicted non-coaxial sand behaviour in undrained torsional shear tests (data from Yoshimine et al. 1998).

Good agreement between the test data and model simulations can be observed.

An important feature of the present model is the non-coaxial flow rule in Equations 12 and 13, resulting naturally by the introduction of an evolving fabric tensor into the yield function and the associative flow rule assumption in the deviatoric stress space.

In a torsional shear test, the radial stress σ_r is always the intermediate principal stress and the radial strain ε_r the intermediate principal strain. In this setting, it is convenient to use the model to explain the non-coaxiality in the z-θ plane (Gao et al. 2014). To elaborate on this point and motivated by the approach in Dafalias et al. (2004), we plot in Figure 2 the variation with deviatoric strain of the difference of the angle $\alpha(\sigma)$ between the direction of the major principal stress σ_1 and the vertical direction, from the angle $\alpha(\varepsilon)$ that the major principal strain ε_1 forms with the vertical direction. Such difference is a measure of non-coaxiality. The simulations match the experimental observation on non-coaxiality qualitatively well. When $\alpha = 0°$ or 90°, there is only change of the principal values of fabric tensor during the development of plastic strain, but no fabric rotation is involved. As such, the two sources of plastic strain increment due to stress and fabric increments will influence its value only, with its direction aligning with the stress direction during the entire loading course. Thus, the predicted sand response is generally coaxial, which is consistent with the experimental observation (Yoshimine et al. 1998). In all the other cases when α is between 0° and 90°, coaxiality is assumed for purely elastic stage (below 0.5% deviatoric strain) due to the employment of isotropic elastic relation. Beyond this elastic stage to a relative low strain level (such as 2%), however, a distinct difference between $\alpha(\varepsilon)$ and $\alpha(\sigma)$ of the order of 4 to 5 degrees on the average is found (Fig. 2), which indicates clearly non-coaxiality. Upon further loading, the fabric tends to rotate towards the direction of stress, and the difference between $\alpha(\varepsilon)$ and $\alpha(\sigma)$ predicted by the model decreases after the peak, and the non-coaxiality will totally disappear at large strain levels.

Figure 3 shows the model simulation for the sand fabric evolution in undrained triaxial extension where static liquefaction occurs. As the fabric tensor is initially triaxial-compression like due to the sample's method of preparation and coaxial with the loading direction, thus, it undergoes only a change of its norm, without any change of its principal directions. In particular the value of its major principal component decreases while the value of its minor principal component increases, which makes the norm F undergo a decrease first until at 7% deviatoric strain. At this point, all components of the fabric tensor are 0 so that a transient isotropic state is observed ($F = A = 0$). As the deformation continues, the original minor component becomes the major one, whilst the original major one turns to be the minor one. The overall degree of anisotropy Fshows a slight rebound from zero (Fig. 3c). The anisotropic variable A increases monotonically from a negative value through zero to a positive one (Fig. 3c). Nevertheless, both A and F reach a very small positive value at static liquefaction where $p = 0$, which is far smaller than their respective critical state value had liquefaction not occurred, which is also observed in the DEM simulations by Li and Li (2009).

4 CONCLUSIONS

A three-dimensional elasto-plastic constitutive model has been proposed to describe the anisotropic behaviour of sand under monotonic loading with fixed principal stress directions. The model constructed within the framework of ACST recently presented by Li and Dafalias (2012), which emphasizes the role of fabric on the characterization of sand response at critical state. The model employs a void-based fabric tensor and a physically-based fabric evolution law to account for the influence of void size and orientation and their change during shear on the sand behaviour including plastic hardening and dilatancy. At the critical state, the fabric tensor has a constant magnitude and

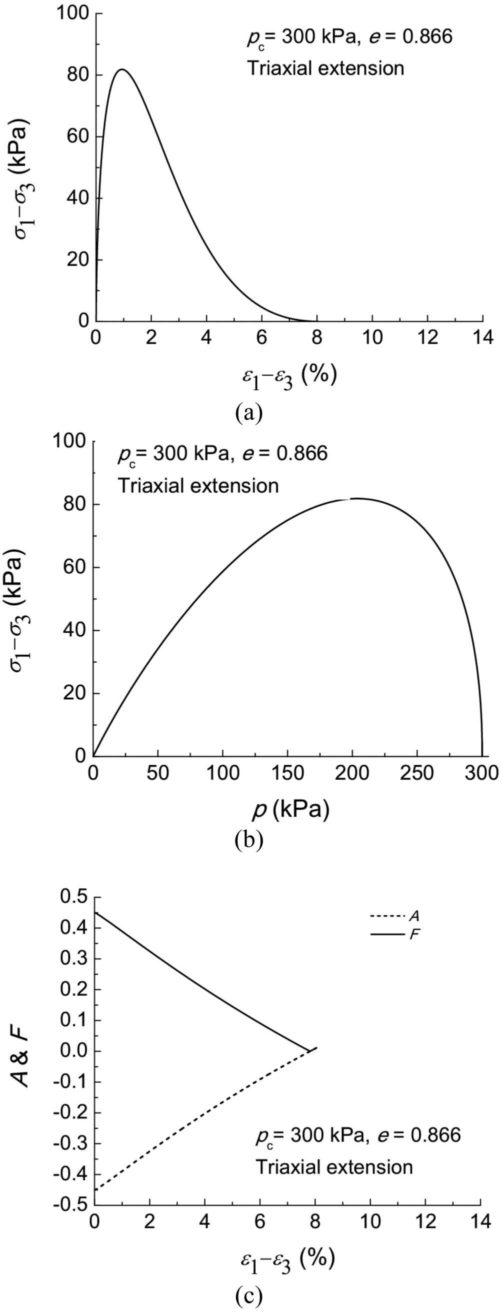

Figure 3. Model simulation for the sand behaviour in undrained triaxial extension and the fabric evolution (static liquefaction occurs).

is co-directional with the loading direction. A non-coaxial but associative flow rule in the deviatoric stress space is used and it can naturally account for the non-coaxial behaviour of initially anisotropic sand samples under monotonic loading.

The model has been used to simulate the undrained test results for the dry-deposited Toyoura sand (Yoshimine et al., 1998) under undrained torsional shear tests with fixed principal stress direction and constant intermediate principal stress variable. The model simulations compare well with the test results.

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