A Theory of Government Procrastination*

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First version: September 2005

This version: July 6, 2009

Abstract

We present a theory to explain government procrastination as a consequence of its present-bias resulting from the political uncertainty in a two-party political system. We show that under a two-party political system the party in office tends to be present-biased. This may lead to inefficient procrastination of socially beneficial policies that carry upfront costs but yield long-term benefits. However, procrastination is often not indefinite even as we consider an infinite-horizon game. There exist equilibria in which the policy is implemented, and in many cases carried out to completion in finite time. The procrastination problem tends to get more serious as the net social benefit of the policy gets smaller. When the net social benefit is large, there is no procrastination problem. When the net social benefit is small, the policy can be procrastinated indefinitely, though there may co-exist equilibria in which the policy is implemented gradually. When the net social benefit is intermediate in magnitude, there is an array of equilibria, all characterized by some form of procrastination, including gradual implementation. The theory predicts that a government with a more predominant party tends to procrastinate less.

JEL Classification: C70, D78, D60

Keywords: present-bias, procrastination, policy implementation

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*We are grateful to Robert Staiger, Jason Saving and participants of the seminars at Australian National University, Chukyo University, City University of Hong Kong, Fukushima University, Hitotsubashi University, Hong Kong University of Science and Technology, Chinese University of Hong Kong, Keio University, Tsukuba University, Seoul National University, Singapore Management University, Waseda University, Yokohama National University, 2005 APTS meeting, ETSG 2005 Seventh Annual Conference, and Spring 2006 Midwest Economic Theory meeting for helpful comments. The work described in this paper is partially supported by grants from the 21st Century Center of Excellence Project on the Normative Evaluation and Social Choice of Contemporary Economic Systems (Japan) and the Research Grants Council of the Hong Kong Special Administrative Region, China [Project No. CityU 1476/05H]. Edwin Lai acknowledges the support of the Department of Economics at Princeton University while writing this paper.

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1 Introduction

People often procrastinate about doing things that yield long-lasting benefits but carry an upfront cost, to the detriment of their long-term interests. Quitting bad habits, such as smoking and drinking, is one prominent example. Other examples include house-cleaning, studying for an examination, and writing a referee report. A recent literature (e.g., Akerlof, 1991 and O'Donoghue and Rabin, 1999) explains this phenomenon by focusing on the existence of present-biased preferences. A present-biased individual's relative preference for payoff at an earlier date over that of a later date gets stronger as these dates approach. These preferences are time-inconsistent, as a cost that appeared to be small yesterday from a present-discount perspective looms large today, while the future benefits appear to be about the same. As a result, a task that appeared to be worth doing today when evaluated as of yesterday becomes unworthy of doing when today arrives, leading to repeated delay. A present-biased individual who is (partially) naive to her own time-inconsistency may procrastinate about completing a task forever, even though it is in her best long-term interest to complete the task immediately.

Similarly, it is often observed that politicians procrastinate about implementing socially beneficial policies that carry upfront costs but yield long-lasting benefits. For example, it is widely believed that the federal government and local governments of the U.S. underinvest in public infrastructure: many bridges need to be repaired, and many stretches of highway need to be renovated. The burst of the dyke in New Orleans in 2005 as a result of hurricane Katrina is a case in point. The public was aware of the potential risk of not strengthening the dyke, and it was clearly a socially beneficial project, yet the government did not act for many years. Another example is that politicians are reluctant to raise income taxes even though it may benefit citizens in the long-run by helping to reduce the government deficit and hence lower the long-term interest rate. The delay of trade liberalization, despite its long-term benefits to the country as a whole, can be explained by the fact that the costs of resource reallocation (such as unemployment of workers) are incurred immediately while social benefits (of lower prices of imported goods for domestic consumers) are spread far into the future. Yet another prominent example of government procrastination is that of pension reform. As Feldstein (2005) states, "[m]any economists and policy analysts acknowledge the long-run advantages of shifting from a pay-as-you go [tax-financed] system to a mixed system [that combines pay-as-you-go benefits with investment-based personal retirement account] but believe that the transition involves unacceptable costs. This is often summarized by saying that the transition generation would have to pay 'double' — once to finance the social security benefits of current retirees and again to save for its own retirement." This might explain why many countries delay pension reform.

In this paper, we provide a theory to explain government procrastination as a consequence of present-bias resulting from the political uncertainty inherent in a two-party political system. We assume that a party has the same time preferences as a typical citizen — which is characterized by geometric discounting — if the party believes it will be in office in every future period. Its discount factor between any two consecutive periods is constant, and its utility function does not give rise to time-inconsistency.[1] However, under a two-party political system, the ruling party becomes present-biased and time-inconsistent. Present-bias arises because a party's probability of getting elected in the future is less than one, and because it puts more weight on the flow of net social benefit resulting from the policy when it is in office than when it is out of office. As a result, the ruling party in a two-party system often procrastinate about implementing socially beneficial policies that carry upfront costs but yield long-term benefits. Specifically, we consider a divisible policy with a positive present discounted value (pdv) of net social benefits, and demonstrate that, depending on the cost of the policy relative to the pdv of future benefits, the government may (i) implement the policy immediately, exactly in accordance with citizens' interests, (ii) procrastinate somewhat, but still implement the policy to completion in finite time, (iii) spread out the implementation over many periods, with the process continuing indefinitely, or (iv) fail to implement any part of the policy.

In a multi-party political system, the policy implementation game that determines when

[1] By making this assumption, we rule out government procrastination resulting from differences in the discount factor between the political parties and the citizenry or among the political parties themselves.

a socially beneficial policy is implemented differs in one fundamental way from a game between the present self and the future selves that determines when a task is carried out. The multi-party policy implementation game is played by the current ruling party against its own future selves as well as the future selves of the rival parties. Nonetheless, we find that there are features of the equilibria that resemble those of a game played by a present self against her own future selves. When the parties are symmetric, we can even interpret the political game as one played by a party's present self against its own future selves.

Indefinite procrastination of socially beneficial policies can sometimes be explained by a model of myopic government who cares more about current constituents and discounts heavily future unborn generations. That is, the government discounts future more heavily than the typical citizen but they both remain time-consistent. In this kind of setting, the government has incentives to procrastinate about implementing a socially beneficial policy indefinitely if and only if the government discounts future sufficiently heavier than the citizens. Since the government remains time-consistent, the policy is either implemented in its entirety immediately or procrastinated indefinitely depending on the government's discount factor. Thus, such models cannot explain why governments sometimes implement a policy only gradually. On the contrary, ours is not a model of myopia. Instead, it is a model of endogenous time-inconsistency of the political parties. A present-biased ruling party may not want to implement the policy now, but may wish a future ruling party would implement the policy; such time-inconsistency never occurs for myopic governments. The outcome of the model is also different from that of myopic government in that there exist equilibria in which, despite certain degree of procrastination, a socially beneficial policy is implemented and carried out to completion in finite time even as we consider an infinite-horizon game. Thus, our analysis reveals the distinction between two sources of procrastination by governments. The first arises from the government being more impatient than the citizens, i.e. a myopic government. The second arises from endogenous present-bias as political parties face uncertainty about the prospect of being elected and put more weight on the flow of net social benefit when they are in office. In this paper, we focus on the second source, which is the more interesting one.

We shall assume that a policy can be partially implemented by a government. For example, a government can choose to partially liberalize the trade regime by cutting only some tariffs, or lowering tariffs somewhat but not all the way to free-trade level. In the case of balancing the budget, a government can choose to reduce the deficit somewhat but not all the way to a balanced budget. Our analysis shows that the possibility of partially implementing the policy enables the government to bypass the fate of indefinite procrastination of the policy even when the net social benefit is small. Seen in this light, this paper identifies a new source of gradualism in the literature on dynamic contribution to a public good, namely the endogenous present-bias of ruling parties inherent in a two-party political system.[2]

Our paper is related to the work of Alesina and Tabellini (1990) and Amador (2003). In their studies of government debt, they argue that the government saves too little, or accumulates too much debt, due to the political uncertainty caused by the two-party system. Amador (2003) observes that the time-inconsistency with which the government is faced is equivalent to the problem faced by a present-biased consumer. In contrast, our paper explains the mechanism through which a ruling party comes to have present-biased preferences in a two-party political system and how this affects the dynamic inefficiency of the policy implementation of the government as one entity. To make the model as general as possible, we have introduced asymmetries in our model: everything else being equal, the parties have asymmetric probabilities of being elected; moreover, the same party has different probabilities of being elected when it is an incumbent as opposed to a non-incumbent.[3] Finally, instead of applying the model to a specific policy issue, we analyze a more general setting, which

[2]Compte and Jehiel (2004), for example, obtain endogenous gradualism in a contribution game by assuming that raising a player's contribution in the negotiation phase increases the other player's outside option value. Each player gradually makes contributions to prevent their respective partner from terminating the game.

[3]In our model, the election outcome is characterized by a Markov process, such that the current ruling party will be re-elected with an exogensous probability between 0 and 1. Moreover, that probability can be different for the two parties. Alesina and Tabellini (1990) and Amador (2003), however, assume that every party has an equal probability of being elected in every election. That is a special case of ours, when the probability of being re-elected equals one half for both parties. Although Alesina and Tabellini (1990) mention in a footnote of their paper that the analysis can be extended to a similar framework to ours, they have not explored how the likelihood of being re-elected affects the government present-bias as much as we do in this paper.

can be further refined for analyzing specific policy issues.

Alesina and Drazen (1991) explain the delay in fiscal stabilization by a game of war of attrition between two heterogeneous socio-economic groups with conflicting distributional objectives. Stabilization is delayed because there is a stalemate in which the groups try to shift the burden of the policy change onto each other. The game in this paper can be viewed partly as a game of war of attrition between the two parties. Each party is reluctant to preside over the initial adjustment period (when the cost of the policy is paid), which citizens dislike. So, each ruling party has incentives to procrastinate, hoping that the other party would implement in the future. As a result, the socially beneficial policy is implemented immediately only if the cost is sufficiently low. Otherwise, there is always some form of procrastination. Procrastination can take the form of one party always procrastinating when in office while the other always implementing when in office, or each party implementing a fraction of the remainder of the policy when in office (if the cost is intermediate), or both parties always procrastinating when in office (if the cost is high). Our difference with Alesina and Drazen (1991) is that there is no need for the two parties to be heterogeneous in any way for procrastination to occur in equilibrium. The above procrastination equilibria exist even when the two parties are perfectly symmetric.

There is one form of procrastination in this model that is not related to war of attrition, but purely the consequence of the present-bias of the parties. It is an equilibrium in which each party, when in office, implements a fraction of the remainder of the policy. As a result, the policy is implemented gradually. The motivation of this form of procrastination is not to shift the burden of the cost of the policy, but to be able to sustain a subgame perfect equilibrium in which all future ruling parties have incentive to implement a fraction of the remainder of the policy. Knowing this, the current ruling party has an incentive to implement a fraction of the remainder of the policy today, even though the party's present discounted utility derived from its own action is negative. This type of equilibrium exists because the current ruling party obtains positive welfare if the policy is implemented some date in the future, but it obtains negative welfare if it implements today.

Our theory predicts that a government with a more predominant party tends to procrastinate less. Thus a government with a overwhelming majority party tends to implement socially desirable reforms more quickly. It also predicts that the predominant party procrastinates less than the predominated party, or that the party that perceives itself to predominate in the future tends to procrastinate less than the one that perceives itself to be predominated in the future. Finally, it predicts that socially desirable policies are often implemented gradually, especially when no party clearly predominate the other. These are all testable hypotheses.

In section 2, we lay down the basic assumptions and setup of the model. In section 3, we show how a two-party political system gives rise to present-bias of the party in office. We consider a socially beneficial policy that carries an upfront cost and yields long-lasting flows of benefits. Given that two parties compete for office in each period, the party currently in office plays a game with all future ruling parties (including its future selves) in choosing the fraction of the policy to be implemented today. In section 4, we explain why there is incentive for a ruling party to procrastinate. In section 5, we compute the subgame perfect equilibria corresponding to different implementation costs. In section 6, we summarize the results and conclude.

2 Preliminaries

There are two political parties, A and B, that seek control of the government. One of them is in office in period $t \in \{0, 1, 2, \cdots\}$. Let each period be a term. Each party discounts future with a discount factor $\delta \in (0, 1)$, which is the same as the discount factor of a representative citizen.

The selection of the party in office in each election is characterized by a Markov process, such that the probability that a party is elected in an election is dependent only on who is currently in office. To be concrete, the probability that party A is re-elected in the next election if it is currently in office is ϕ^A, while the probability that party B is re-elected if it is currently in office is ϕ^B. Since there are only two parties, the probability that one party wins is equal to the probability that the other party loses. Therefore, ϕ^A and ϕ^B are the only two parameters needed to fully describe the Markov process. We shall analyze this Markov process in detail in the next section. We have assumed for simplicity that the probability that a party is elected is independent of how the policy is implemented by the party or its rival. This is clearly a limitation. But this assumption allows us to focus on the issue of interest and to present our main findings transparently. Moreover, it enables us to conduct a simple analysis concerning how party predominance affects the policy implementation outcome when such predominance is exogenously given.[4]

The policy that we consider is about implementing a policy that involves an immediate implementation cost of c but generates a constant benefit flow of 1 in the current period and every future period. We assume that the policy is divisible in the sense that a ruling party can choose to implement only a fraction of the policy in its term so that a fraction a_t of the policy undertaken in period t poses an upfront cost $a_t c$ to society while generating benefit flows of a_t in each future period. We assume that $1/(1-\delta) > c$, so the policy is worth implementing from the citizens' point of view.

The flow of utility enjoyed by citizens in period t is assumed to be equal to the flow of net social benefit resulting from the policy in that period, which is given by

$$u_t = \sum_{k=0}^{t} a_k - a_t c.$$

The first term on the right-hand side shows the flow of benefit that society enjoys in period t from the fraction of the policy that has been implemented, whereas the second term represents the flow of cost that society incurs from the part of the policy implemented in period t. Therefore, citizens's welfare function at time t is

$$W_t = \sum_{k=0}^{\infty} \delta^k u_{t+k}.$$

We assume that the party in office in period t places a (normalized) weight of one on the flow of net social benefit in period t, and so its flow of utility in period t equals u_t, while

[4]It is often the case that party predominance is quite exogenous for historical reasons. Examples are the Liberal Democratic Party in post-war Japan, the People's Action Party in post-independence Singapore, and the Congress Party in post-independence India.

the opposition party puts a weight of $\alpha \in [0, 1]$ on the flow of net social benefit in the same period.[5] In other words, a party puts more weight on the flow of net social benefit when it is in power than when it is not. This differential weighting is motivated by the presumption that the ruling party's welfare function is a weighted sum of the citizens's welfare and its own welfare (α is the weight put on social welfare and $1 - \alpha$ is the weight put on the party's welfare). We suppose the ruling party derives some flow of private benefits (costs) spilled over from a positive (negative) flow of net social benefit during its term. Moreover, the flow of private benefits, which can be negative, is assumed to be proportional to the flow of net social benefit. For example, when the ruling party presides over a larger positive (negative) flow of net social benefit, citizens are better (worse) off, and are therefore more (less) willing to accommodate higher government spending during its term. This in turn increases (decreases) the amount of resources available to the ruling party to pursue its own agenda during that period.[6]

3 Endogenous Present-Bias

In this section, we show that in a two-party political system, the party in office will possess present-biased preferences. By present-bias, we mean that the discount rate for the next-period flow of utility is greater in the current period than that in any other future period. In other words, the ruling party of a given period puts a disproportionately high weight on the current flow of utility. We also show that if an incumbent advantage (which is defined shortly) exists, the party in office will possess a utility function with generalized hyperbolic discounting. By generalized hyperbolic discounting, we mean one such that the discount rate between two consecutive periods is higher between today and tomorrow than between any

[5]The model can easily be accommodated to the case where the parties have different values of α. We assume that they have the same value of α only to simplify the exposition.

[6]Another way to motivate why a party puts more weight on the flow of net social benefit when in office is that the ruling party not only cares about the flow of net social benefit as a typical citizen, but it also cares more about it because delivering a larger flow of net social benefit while in office enhances its political status. On the contrary, while the opposition party cares about the flow of net social benefit as a typical citizen, it cares less about it because it treats the success of the ruling party as unfavorable, as it undermines its political status.

other two future consecutive periods; moreover, the discount rate diminishes when the two consecutive periods are further into the future. Thus preferences with generalized hyperbolic discounting are present-biased.[7]

Let p_k^i denote the probability that party i currently in office will also be in office k periods later, and consider the case in which the flow of net social benefit u_{t+k} in every future period $t+k$ does not depend on which party is in office in period $t+k$. Then, it follows from our discussion above that the welfare function for party i when it is in office in period t is given by

$$U_t^i = \sum_{k=0}^{\infty} \delta^k [p_k^i + (1-p_k^i)\alpha] u_{t+k} \equiv \sum_{k=0}^{\infty} \beta_k^i u_{t+k} \tag{1}$$

In other words, the utility flow accrued to ruling party i is equal to the flow of utility to the citizens in each period, but the party's discount factor differs from that of the citizens in each period because of political uncertainty and selfishness of the party, as discussed above. Then, U_t^i exhibits present-bias if $\beta_1^i/\beta_0^i < \beta_{k+1}^i/\beta_k^i \;\; \forall\, k$, and it exhibits generalized hyperbolic discounting if the ratio of the two consecutive discount functions β_{k+1}^i/β_k^i weakly increases with k.[8] Recall that ϕ^i is the probability that party i is re-elected when it is currently in office. Define $s \equiv \phi^A + \phi^B$. Clearly, $s \in (0, 2)$. We shall show below that U_t^i exhibits generalized hyperbolic discounting if $2 > s \geq 1$.

For concreteness, let us suppose for now that party A is in office in the current period. We derive below the probability that the current ruling party will also be in office k periods later and show that it converges to a steady state probability as k becomes large. First, note

[7] The psychological basis for present-bias in an individual's preferences is that the distinction between two consecutive periods is more salient between today ($t = 0$) and tomorrow ($t = 1$) than between any other two consecutive future periods, and the distinction becomes less so when the two consecutive periods are further into the future. This is because today is absolutely certain while all future days are uncertain. Akerlof (1991) gives an excellent discussion about the salience of the present for a present-biased individual.

[8] The instantaneous discount rate of the "usual" exponential discount function $\beta_e(t) \equiv e^{-rt}$ in continuous time models is given by $-\beta_e'(t)/\beta_e(t) = r$, whereas that of hyperbolic discount function $\beta_h(t) \equiv (1+\alpha t)^{-\gamma/\alpha}$ is given by $-\beta_h'(t)/\beta_h(t) = \gamma/(1+\alpha t)$ that decreases with t (for hyperbolic discounting, see Loewenstein and Prelec, 1992, who call it *generalized* hyperbolic discounting contrary to our terminology). Phelps and Pollak (1968) develop an intertemporal utility function of the form: $U_t = u_t + \beta \sum_{k=1}^{\infty} \delta^k u_{t+k}$ (where $0 < \beta < 1$ and $0 < \delta < 1$) to capture imperfect altruism for future generations. Laibson (1997) introduces this utility function with quasi-hyperbolic discounting to behavioral economics in order to capture important properties of hyperbolic discounting. Note that quasi-hyperbolic discounting is a special case of generalized hyperbolic discounting as β_{k+1}/β_k weakly increases with k ($\beta_1/\beta_0 = \beta\delta$ and $\beta_{k+1}/\beta_k = \delta$ for $k \geq 1$).

that the probability that party A will be in office $k+1$ periods later can be linked to the probability that party A is in office k periods later, as follows:

$$\begin{aligned} p_{k+1}^A &= \phi^A p_k^A + (1-\phi^B)(1-p_k^A) \\ &= (1-\phi^B) + (s-1)p_k^A, \end{aligned}$$

with $p_0^A = 1$. When $s \neq 1$, we can solve this difference equation explicitly to obtain

$$p_k^A = \frac{(1-\phi^B) + (1-\phi^A)(s-1)^k}{2-s}. \tag{2}$$

When $s = 1$, it is obvious that $p_0^A = 1$ and $p_k^A = \phi^A$ for $k \geq 1$.

Define $p^A \equiv \lim_{k\to\infty} p_k^A$. Then, it is clear that $p^A = \frac{\phi^A-(s-1)}{2-s}$. For $s \neq 1$, we can rewrite (2) as

$$p_k^A = p^A + (1-p^A)(s-1)^k \text{ for } k \geq 0\ . \tag{3}$$

When $s = 1$, we have $p_0^A = 1$ and $p_k^A = p^A$ for $k \geq 1$. Similarly, we have

$$p_k^B = 1 - p^A + p^A(s-1)^k \text{ for } k \geq 0. \tag{4}$$

As we see from (3) and (4) (together with $0 < s < 2$) that p_k^A and p_k^B approach p^A and $1-p^A$, respectively as k becomes large. That is, p^A is the steady state probability that party A is in office. Without loss of generality, we assume that $p^A \geq 1/2$, or equivalently $\phi^A \geq \phi^B$. That is, we assume that party A is a (weakly) predominant party.

In fact, $s-1$ measures the degree of incumbent advantage in an election. The probability that party i wins the election generally depends on whether or not it is currently in office. The winning probability is greater when it is currently in office than otherwise if and only if $\phi^i > 1-\phi^j$, where $j \neq i$, or equivalently $s > 1$. That is, the incumbent has an advantage in the next election if $s > 1$.[9] It is in this sense that $s-1$ represents the degree of incumbent advantage. It follows that there is incumbent disadvantage if $0 < s < 1$, and neither incumbent advantage nor disadvantage if $s = 1$.

[9]In fact, being an incumbent boosts the probability that a party is elected by $\phi^i - (1-\phi^j)$ which is equal to $s-1$.

As we see from (3), the probability that party A is in office decreases over time from $p_0^A = 1$ and converges to p^A. The proof consists of three parts. (1) When $1 < s < 2$, incumbent A has an advantage in the next election, but this advantage diminishes over time. The case where party B is currently in office is similar; the probability that party B is in office decreases over time from $p_0^B = 1$ to $1 - p^A$. (2) If $s = 1$, there is neither incumbent advantage nor disadvantage. The probability that a party is in office remains constant over time, and this probability is independent of whether or not the party is currently in office: $p_k^A = 1 - p_k^B = p^A$ and $p_k^B = 1 - p_k^A = 1 - p^A$ for any $k \geq 1$. (3) Finally, there exists an incumbent disadvantage when $0 < s < 1$. The probability that party A is in office k periods later, p_k^A fluctuates around p^A, and converges to p^A. Similarly, if B is currently in office, the probability that party B is in office fluctuates around $1 - p^A$ and converges to $1 - p^A$.

To show that the party in office has present-biased preferences, consider a stream of flows of net social benefit $\{u_{t+k}\}_{k=0}^{\infty}$. Recalling that party i discounts the flow of net social benefit by a factor α when it is not in office, we write the expected welfare for ruling party i in period t as $U_t^i = \sum_{k=0}^{\infty} \beta_k^i u_{t+k}$, where $\beta_k^i = \delta^k[p_k^i + (1 - p_k^i)\alpha]$. For concreteness, let us consider the case where party A is in office in period 0. Then, the discount function for party A can be written as

$$\begin{aligned} \beta_k^A &= \delta^k[p_k^A + (1 - p_k^A)\alpha] \\ &= \delta^k\{\alpha + (1 - \alpha)[p^A + (1 - p^A)(s - 1)^k]\}. \end{aligned} \tag{5}$$

The utility function for the incumbent party i exhibits generalized hyperbolic discounting if $\beta_{k+1}^i / \beta_k^i$ weakly increases with k. It directly follows from (5) that

$$\frac{\beta_{k+1}^A}{\beta_k^A} = \delta \left[\frac{\alpha + (1 - \alpha)\{p^A + (1 - p^A)(s - 1)^{k+1}\}}{\alpha + (1 - \alpha)\{p^A + (1 - p^A)(s - 1)^k\}} \right]; \tag{6}$$

we have similar expression for $\beta_{k+1}^B / \beta_k^B$. As Figure 1 indicates, this ratio of discount functions changes with k differently depending on the value of s. First, it can be readily verified from (6) that if $2 > s > 1$, then $\beta_{k+1}^i / \beta_k^i$ increases with k and converges to δ as k tends to infinity. Thus, the ruling party's utility function exhibits generalized hyperbolic discounting in this case. If $0 < s < 1$, on the other hand, $\beta_{k+1}^i / \beta_k^i$ fluctuates around δ as k increases, such that it is less than δ when k is even, is greater than δ when k is odd, and converges to δ as k tends to infinity. Moreover, β^i_{k+1}/β^i_k takes on the smallest value when $k = 0$, which implies that the discount rate is greatest in the current period, i.e., the ruling party has present-biased preferences as in the case where $2 > s > 1$.

Finally, if $s = 1$, it follows from $\beta^A_0 = 1$ and (6) that $\beta^A_1/\beta^A_0 = \delta[\alpha + (1-\alpha)p^A] < \delta$ and $\beta^A_{k+1}/\beta^A_k = \delta$ for $k \geq 1$, and similarly for party B. Therefore, each party's utility function exhibits quasi-hyperbolic discounting (Laibson, 1997; see also footnote 8). The ruling party discounts the flow of net social benefit in the next period more heavily than the discounting brought about by the discount factor δ as it will be out of office with a positive probability. Since the probability of being in office is the same for all future periods whether or not a party is currently in office (i.e., the party never enjoys incumbent advantage nor disadvantage in future elections), discounting between any two future consecutive periods is stationary.

In a similar multi-party political environment as ours, Amador (2003) shows that if all political parties including the incumbent party have equal probabilities of being elected in the next election, the preferences of the incumbent party is characterized by quasi-hyperbolic discounting. His model therefore corresponds to the case where $s = 1$ and $p^A = 1/2$ in our model.[10]

We record the above findings in the following proposition.

Proposition 1 *A two-party political system leads to present-biased preferences of the party in office. The preferences of the party in office are characterized by generalized hyperbolic discounting in the presence of a (weak) incumbent advantage in elections.*

We have shown that the party in office is present-biased, regardless of the degree of the incumbent advantage. Present-bias causes time-inconsistency in the ruling parties' decision making whether or not the parties are symmetric (i.e., $p^A = 1/2$) in which case they have exactly the same preferences when in office. To make our points more transparent, we henceforth assume that $2 > s \geq 1$. In this case, each party's utility function exhibits

[10]Our argument can easily be generalized to the case of multi-party political system with more than two parties. We demonstrate our argument in the case of two parties to avoid the discussion of issues such as coalition formation to gain a majority, which are not of central interest in our analysis.

generalized hyperbolic discounting, which plays an important role especially in the existence of the equilibrium with gradual policy implementation when the cost of the policy is relatively high.

Before turning to the issue of policy implementation by present-biased parties, we investigate how the basic parameters affect the degree of present-bias. First, it is readily verified from (6) that β^A_{k+1}/β^A_k increases with α for any k. That is, the higher the weight a party places on the flow of net social benefit when it is not in office, the less present-biased are its preferences. In the extreme case where $\alpha = 1$, we have $\beta^i_{k+1}/\beta^i_k = \delta$, i.e., each party i's preferences exhibit geometric discounting, and there is no present-bias. Next, an increase in party A's predominance will make party A less present-biased, and make the predominated party B more present-biased. This can be seen from the observation that β^A_{k+1}/β^A_k increases with p^A and β^B_{k+1}/β^B_k decreases with p^A (which can also be readily verified), with s held constant. Indeed, when $1 < s < 2$, the predominant party A is less present-biased than the predominated party B, i.e., $\beta^A_{k+1}/\beta^A_k > \beta^B_{k+1}/\beta^B_k$ for any k.

4 Temptation to Procrastinate

In this section we try to gain some intuition about the decision-making of the ruling parties. It has been shown in the literature that an individual with a quasi-hyperbolic utility function exhibits time-inconsistent behavior, which includes inefficient procrastination of beneficial tasks that carry upfront costs but generate long-lasting future streams of benefits (see, for example, O'Donoghue and Rabin, 1999). In the current setting, the ruling party prefers the other party, when in office, to implement the policy and bear the upfront implementation cost. The policy implementation game is a war of attrition; each party has an incentive to wait, hoping that the other party would concede (i.e. implement and bear the policy cost). As a consequence, the party in office will have a present-biased utility function. Therefore, it is also faced with a time-inconsistency problem, and we expect that it may procrastinate.

Our analysis shows that procrastination occurs under certain conditions, and the problem gets worse as implementation cost gets higher. However, even when it does happen, procrastination needs not be indefinite even as we consider an infinite-horizon game. Although the government sometimes procrastinates implementing socially beneficial policies, there exist equilibria in which the policy is implemented, and may be carried out to completion in finite time, especially when the cost is low. Specifically, we show that (i) the policy is entirely implemented immediately in period 0 if the cost of the policy is small; (ii) there may be some finite delay in implementing the policy or the policy is implemented gradually over many periods of time if the cost is in the intermediate range; and (iii) if the cost is high, the policy may never be implemented, but there may also co-exist other equilibria in which the policy is implemented gradually. The equilibrium with gradual policy implementation when the policy implementation cost is high exists precisely because the party in office possesses hyperbolic discounting.

We shall show that there is a temptation for the current ruling party to procrastinate due to its hyperbolic discounting. Suppose we ignore for now the divisibility of the policy. The expected present discounted utility of ruling party i (evaluated at $t = 0$) based on the anticipation that the entire policy is implemented by whoever is in office (not necessarily party i) t periods later is given by

$$X_t^i \equiv \sum_{k=0}^{\infty} \beta_{t+k}^i - \beta_t^i c.$$

Therefore, ruling party i at period 0 (weakly) prefers having the policy implemented in period 0 to having it done in period 1 if and only if

$$\begin{aligned} & X_0^i \geq X_1^i \\ \Leftrightarrow \;\; & \beta_0^i \geq (\beta_0^i - \beta_1^i)c \\ \Leftrightarrow \;\; & \frac{\beta_1^i}{\beta_0^i} \geq \frac{c-1}{c}. \end{aligned} \tag{7}$$

The second inequality is easy to interpret: If the ruling party in period 0 knows that the policy will be implemented by whoever is in office in period 1, it prefers to implement the policy in its entirety immediately if and only if the reduction in benefit by procrastinating, β_0^i, is at least as high as the reduction in cost by doing so, $(\beta_0^i - \beta_1^i)c$. If c is large enough that

$\beta_1^i (= \beta_1^i/\beta_0^i) < (c-1)/c$, both parties (whenever they are in office) want to procrastinate. Since $p^A > 1/2$, party B has stonger incentive to procrastinate than party A.

If neither party discounts the flow of net social benefit when it is out of office (i.e., $\alpha = 1$), then $\beta_t^i = \delta^t$ for all $t \geq 0$ and for $i = A, B$. In that case, inequality (7) always holds, as it is reduced to $1/(1-\delta) \geq c$. Thus, the ruling party in period 0 prefers implementing to procrastinating. Note that, since $\beta_t^i = \delta^t$, the ruling party's welfare function is exactly the same as that of the citizens. Therefore, in this case, the government's action maximizes the welfare of the citizens. We summarize this finding in the following proposition.

Proposition 2 *Suppose that neither party discounts the flow of net social benefit when it is out of office, i.e.,* $\alpha = 1$*, then neither party would procrastinate about implementing a socially beneficial policy when it is in office.*

On the other hand, if the parties discount the flow of net social benefit when they are out of office (i.e., $\alpha < 1$), then procrastinating may be preferable for the ruling party since, by doing so, the reduction in cost can outweigh the loss in benefit. We start the analysis from the following lemmas.

Lemma 1 *Considering only stationary pure strategies, if* $X_0^i > 0$*, then ruling party* i *can gain from procrastinating only if party* j $(\neq i)$ *always implements when in office. In other words, provided that* $X_0^i > 0$*, given that party* j *always procrastinates, the best response of ruling party* i *is to always implement.*

Proof. If $X_0^i > 0$, then ruling party i cannot rely on its future self to implement the policy when in office, precisely because of time-inconsistency. When a future period comes, even if party i is in office, it would face exactly the same situation as in period 0, and so it would procrastinate based on the same reasoning as in period 0. **Q.E.D.**

Lemma 2 *Considering only stationary pure strategies, if* $X_0^i < 0$*, then ruling party* i *always procrastinates.*

Proof. Since perpetual procrastinating yields zero welfare, it is better than implementing, which yields negative welfare. **Q.E.D.**

Lemma 3 *Considering only stationary pure strategies, if $\beta_1^i > (c-1)/c$, then ruling party i always implements regardless of party j's implementation strategy.*

Proof. According to equation (7), if $\beta_1^i/\beta_0^i = \beta_1^i > (c-1)/c$, "always implement" is a dynamically consistent strategy for party i, given that party j always implements: Given that party i would implement whenever it is in office in the future, party i is better off implementing immediately if it is currently in office. **Q.E.D.**

Lemma 4 *Considering only stationary pure strategies, if $\beta_1^i < (c-1)/c$ but $X_0^i > 0$, ruling party i always procrastinates given that party j always implements; and ruling party i always implements given that party j always procrastinates.*

Proof. If $\beta_1^i < (c-1)/c$, then given that party j always implements when in office, it is not optimal for party i to adopt the stationary strategy of always implementing when in office. Therefore, the only stationary pure strategy for party i is to always procrastinate when in office.[11] Finally, if $\beta_1^i < (c-1)/c$ and $X_0^i > 0$, then, from lemma 1, ruling party i always implements given that party j always procrastinates. **Q.E.D.**

5 Subgame Pefect Equilibria

5.1 Non-Cooperative Stationary Subgame Perfect Equilibrium

Lemmas 1 through 4 basically provide the intuition that if we consider only stationary strategies then party i would (1) always implement when $\beta_1^i > (c-1)/c$; (2) when $\beta_1^i < (c-1)/c$ and $X_0^i > 0$, (a) always implements when party j always procrastinates; (b) always procrastinates when j always implements; (3) always procrastinates when $X_0^i < 0$.

[11] The strategy is dynamically consistent: If the utility from procrastinating is higher than from implementing today given that the calculation is based on the assumption that party i would implement in the next period (if it is in office), then the value from procrastinating today would be even higher given that the calculation is based on the assumption that party i procrastinates again in the next period (if in office), as the party would face the same situation tomorrow as today.

Recall that the welfare of ruling party i at $t = 0$ if it implements the policy immediately is $X_0^i = \sum_{k=0}^{\infty} \beta_k^i - c = 1 - c + \sum_{k=1}^{\infty} \beta_k^i$. Therefore, $X_0^i < 0 \Longleftrightarrow (c-1)/c > \sum_{k=1}^{\infty} \beta_k^i / \sum_{k=0}^{\infty} \beta_k^i$.

Consequently, lemmas 1 through 4 boils down to the following: party i would

(1) always implement when $(c-1)/c < \beta_1^i$;

(2) when $\beta_1^i < (c-1)/c < \sum_{k=1}^{\infty} \beta_k^i / \sum_{k=0}^{\infty} \beta_k^i$, (a) always implements when party j always procrastinates; (b) always procrastinates when j always implements;

(3) always procrastinates when $\sum_{k=1}^{\infty} \beta_k^i / \sum_{k=0}^{\infty} \beta_k^i < (c-1)/c$.

Therefore, its implementation strategy depends on the value of $(c-1)/c$.

This subsection formally derives non-cooperative stationary subgame perfect equilibria that confirm the intuition behind lemmas 1 through 4. Subsection 5.2 derives a non-cooperative subgame perfect equilibrium with gradual implementation. Subsection 5.3 analyzes the effects of party predominance. Subsection 5.4 shows that even in the case where the cost is so high that there exists no non-cooperative equilibrium with successful implementation of the policy, there may exist a "cooperative" equilibrium (with a trigger strategy and a possible punishment strategy) in which the policy is gradually implemented.

Mixed Strategy Equilibrium

We derive the condition for the existence of a mixed-strategy equilibrium. A mixed-strategy equilibrium exists when each ruling party derives positive utility from implementing the policy in its entirety immediately but would gain from procrastinating if it knows that the other party would implement when in office next period. In such situations, an equilibrium exists such that all ruling parties randomize their policy decisions, and each party is made indifferent between implementing or procrastinating when in office. This corresponds to case (2) mentioned above, namely when $\beta_1^i < (c-1)/c < \sum_{k=1}^{\infty} \beta_k^i / \sum_{k=0}^{\infty} \beta_k^i$.

Define $\tilde{\beta}_k^i = \delta^k [1 - p_k^j + p_k^j \alpha]$ (where $j \neq i$) as party i's discount function k periods later given that it is not in office today. Define also $\tilde{X}_0^i = \alpha(1-c) + \sum_{k=1}^{\infty} \tilde{\beta}_k^i$ as the welfare of party i when party j implements the policy immediately given that j is in office at $t = 0$. Let σ^i denote the stationary probability that ruling party i implements the entire policy given that it has not been implemented. Given that the policy has not been implemented, let V^i ($\tilde{V}^i$)

denote the expected welfare of party i (which may or may not be in office) at the beginning of each period when party i (j) was in office last period.

Then, in the mixed-strategy equilibrium, V^A and $\tilde{V}^A$ must simultaneously satisfy

$$V^A = \phi^A[\sigma^A X_0^A + (1-\sigma^A)\delta V^A] + (1-\phi^A)[\sigma^B \tilde{X}_0^A + (1-\sigma^B)\delta \tilde{V}^A], \tag{8}$$

$$\tilde{V}^A = (1-\phi^B)[\sigma^A X_0^A + (1-\sigma^A)\delta V^A] + \phi^B[\sigma^B \tilde{X}_0^A + (1-\sigma^B)\delta \tilde{V}^A]. \tag{9}$$

In the mixed-strategy equilibrium, ruling party A is indifferent between implementing and procrastinating, i.e., $X_0^A = \delta V^A$. Substituting this into (9) and solving it for $\tilde{V}^A$, we obtain

$$\tilde{V}^A = \frac{(1-\phi^B)X_0^A + \phi^B \sigma^B \tilde{X}_0^A}{1-\delta\phi^B(1-\sigma^B)}.$$

Then, we substitute this expression and $X_0^A = \delta V^A$ into (8) to obtain

$$V^A = \frac{[\phi^A - \delta(s-1)(1-\sigma^B)]X_0^A + \sigma^B(1-\phi^A)\tilde{X}_0^A}{1-\delta\phi^B(1-\sigma^B)}. \tag{10}$$

We apply $X_0^A = \delta V^A$ one more time to equation (10) to get the probability of implementation by ruling party B that renders party A indifferent between implementing and procrastinating when in office:

$$\sigma^{B*} = \frac{1-\delta s - \delta^2(s-1)}{\delta\left[(1-\phi^A)\frac{\tilde{X}_0^A}{X_0^A} - \phi^B\right] + \delta^2(s-1)}. \tag{11}$$

Similarly, we obtain the corresponding probability to be chosen by A to make B indifferent between implementing and procrastinating when in office:

$$\sigma^{A*} = \frac{1-\delta s - \delta^2(s-1)}{\delta\left[(1-\phi^B)\frac{\tilde{X}_0^B}{X_0^B} - \phi^A\right] + \delta^2(s-1)}.$$

In the mixed-strategy equilibrium, σ^{B*} is chosen by ruling party B so that party A is indifferent between implementing and procrastinating when in office. Thus, in situations where ruling party A's incentive to procrastinate decreases, σ^{B*} must be increased to preserve this indifference. Consequently, if σ^{B*} calculated in (11) is greater than 1, ruling party A will always implement regardless of B's implementation strategy. On the other hand, if $\sigma^{B*} < 0$, ruling party A will procrastinate regardless of the implementation strategy of party B.

Relationship Between Equilibrium Outcome and Implementation Cost

The top panel of Figure 2 illustrates the parties' implementation strategies. It is assumed that $\phi^A > \phi^B$, i.e., party A is a predominant party. The above analysis shows that for party i (where $i = A, B$ and $j \neq i$), when the value of $(c-1)/c$ is in the range marked by $0 < \sigma^{i*} < 1$, ruling party i's best response given j's strategy can take any of the following three alternatives: (1) ruling party i randomizes if it is made indifferent between implementing and procrastinating by suitable choice of $\sigma^{j*} \in (0,1)$ by j, (2) ruling party i always implements the policy if party j always procrastinates when in office, and (3) ruling party i always procrastinates if j always implements when in office. Knowing these strategies, we can delineate the equilibria according to the value of $(c-1)/c$ as follow. The delineation of the equilibria is shown in the lower panel of Figure 2. There are five types of equilibria:

(i) When $(c-1)/c < \beta_1^i$, there is a unique (stationary) pure strategy equilibrium in which A and B both implement;

(ii) when $\beta_1^B < (c-1)/c < \beta_1^A$, there is a unique (stationary) pure strategy equilibrium in which A implements and B procrastinates;

(iii) when $\beta_1^A < (c-1)/c < \sum_{k=1}^{\infty} \beta_k^B / \sum_{k=0}^{\infty} \beta_k^B$, there are multiple stationary equilibria — there are at least three equilibria: (a) both randomize, (b) A implements and B procrastinates, (c) A procrastinates and B implements;

(iv) when $\sum_{k=1}^{\infty} \beta_k^B / \sum_{k=0}^{\infty} \beta_k^B < (c-1)/c < \sum_{k=1}^{\infty} \beta_k^A / \sum_{k=0}^{\infty} \beta_k^A$, there is a unique (stationary) pure strategy equilibrium in which A implements and B procrastinates, just like in (ii);

(v) when $\sum_{k=1}^{\infty} \beta_k^A / \sum_{k=0}^{\infty} \beta_k^A < (c-1)/c$, there is a unique (stationary) pure strategy equilibrium in which both procrastinate.

We summarize these findings in the following proposition.

Proposition 3 *If the implementation cost of the policy is small, the policy is immediately implemented despite the fact that both parties are present-biased. If the cost is high, neither party implements the policy. If the cost is in the intermediate range, some delay in implementation is expected. The delay may arise because one of the two parties always procrastinates when in office, or because both parties mix their decision as to whether or not* *they implement the policy when they are in office.*

To demonstrate that the range of $(c-1)/c$ indicated in Equilibrium Type (iii) above indeed supports multiple equilibria, it proves useful to specialize to $s = 1$ and $\alpha = 0$, i.e. there exists neither incumbent advantage nor disadvantage, and the parties do not care about social welfare when they are not in office. This helps to simplify the exposition without losing generality.

Specialization to $s = 1$ and $\alpha = 0$

In this case, we have $p_k^A = 1 - p_k^B = p^A$ and $p_k^B = 1 - p_k^A = 1 - p^A$ for $k \geq 1$. Then, we have

$$X_0^A = 1 - c + \frac{\delta p^A}{1-\delta}, \tag{12}$$

$$\tilde{X}_0^A = \frac{\delta p^A}{1-\delta}. \tag{13}$$

We substitute the above equations and $s = 1$ and $\alpha = 0$ into (11) to obtain

$$\sigma^{B*} = \frac{\delta p^A - (1-\delta)(c-1)}{\delta(1-p^A)(c-1)}. \tag{14}$$

It is readily verified that σ^{B*} increases if c decreases or p^A increases. It is necessary for σ^{B*} to increase to reduce ruling party A's incentive to implement the policy when either one of these pro-implementation forces strengthens.

Delineation of Equilibria when $s = 1$ and $\alpha = 0$

In the discussion above, we note that if σ^{B*} is calculated from (11) and if $X_0^A = \delta V^A$ is assumed, then $\sigma^{B*} > 1$ signifies that party A always implements regardless of party B's implementation strategy, and $\sigma^{B*} < 0$ signifies that party A always procrastinates regardless of party B's strategy. Now, we derive the conditions under which $\sigma^{B*} > 1$ and $\sigma^{B*} < 0$, respectively. It follows directly from (14) that $\sigma^{B*} > 1$ is equivalent to

$$1 - c + \frac{\delta p^A}{1-\delta} > \frac{\delta}{1-\delta}(1-p^A)(c-1). \tag{15}$$

which can be written as

$$\frac{c-1}{c} < \delta p^A = \beta_1^A. \tag{16}$$

Under this circumstance, according to lemma 3, ruling party A always implements regardless of party B's implementation strategy, which means that $X_0^A > \delta V^A$ even when $\sigma^{B*} = 1$.[12]

On the other hand, it follows from (14) that $\sigma^{B*} < 0$ is equivalent to

$$1 - c + \frac{\delta p^A}{1-\delta} < 0,$$

which is equivalent to $X_0^A < 0$, i.e. implementing the policy confers negative welfare on ruling party A. This inequality can be rewritten as

$$\frac{c-1}{c} > \frac{\delta p^A}{1-\delta(1-p^A)}. \tag{17}$$

Under this circumstance, ruling party A procrastinates regardless of party B's implementation strategy.

We can conduct a similar analysis for ruling party B and obtain

$$\sigma^{A*} = \frac{\delta(1-p^A) - (1-\delta)(c-1)}{\delta p^A(c-1)}.$$

Ruling party B always implements the policy regardless of party A's implementation strategy if

$$\frac{c-1}{c} < \delta(1-p^A) = \beta_1^B,$$

whereas ruling party B always procrastinates regardless of A's implementation strategy if

$$\frac{c-1}{c} > \frac{\delta(1-p^A)}{1-\delta p^A},$$

which is equivalent to $X_0^B < 0$.

The delineation of equilibria under the special case $s = 1$ and $\alpha = 0$ is shown in the lower panel of Figure 2.

[12] To proof this, note that when $\sigma^{B*} = 1$, we have from (10) that $V^A = p^A X_0^A + (1-p^A)\tilde{X}_0^A$. Using this equality, and equations (12) and (13), we obtain $V^A = p^A\left(1 - c + \frac{\delta p^A}{1-\delta}\right) + (1-p^A)\frac{\delta p^A}{1-\delta}$ so that $V^A - X_0^A = (1-p^A)(c-1)$. Therefore, the right hand side of (15) is equal to $\delta(V^A - X_0^A)/(1-\delta)$. Note that the left hand side of (15) is equal to X_0^A from (12). Therefore, (15) is equivalent to $X_0^A > \delta(V^A - X_0^A)/(1-\delta)$, which is equivalent to $X_0^A > \delta V^A$. Hence, (16) is equivalent to saying that $X_0^A > \delta V^A$ at $\sigma^{B*} = 1$.

5.2 Non-cooperative Equilibrium With Gradual Implementation

The existence of mixed strategy equilibrium leads us to suspect that if the policy is divisible then each party may be willing to implement a fraction of the policy when in office given that the other does the same. Indeed, there exists an equilibrium with gradual implementation of the policy if the implementation cost is in the intermediate range where the mixed-strategy equilibrium exists [Type (iii) in subsection 5.1 and in Figure 2]. This "gradual implementation equilibrium" has a one-to-one correspondence with the mixed-strategy equilibrium.

Let us consider a stationary strategy profile such that whenever party i is in office, it implements a fraction a^i of the remainder of the policy of size $\theta \in (0, 1]$. Then, ruling party A's expected welfare when A was in office in the last period and that when B was in office in the last period can be written, respectively, as functions of θ:

$$V^A(\theta) = (1-q^A)[a^A\theta X_0^A + \delta V^A((1-a^A)\theta)] + q^A[a^B\theta \tilde{X}_0^A + \delta \tilde{V}^A((1-a^B)\theta)],$$
$$\tilde{V}^A(\theta) = q^B[a^A\theta X_0^A + \delta V^A((1-a^A)\theta)] + (1-q^B)[a^B\theta \tilde{X}_0^A + \delta \tilde{V}^A((1-a^B)\theta)].$$

Let us guess that $V^A(\theta)$ and $\tilde{V}^A(\theta)$ are linear such that $V^A(\theta) = \theta v^A$ and $\tilde{V}^A(\theta) = \theta \tilde{v}^A$ where v^A and $\tilde{v}^A$ are time-invariant. Then, these equations can be rewritten as

$$v^A = (1-q^A)[a^A X_0^A + (1-a^A)\delta v^A] + q^A[a^B \tilde{X}_0^A + (1-a^B)\delta \tilde{v}^A], \tag{18}$$
$$\tilde{v}^A = q^B[a^A X_0^A + (1-a^A)\delta v^A] + (1-q^B)[a^B \tilde{X}_0^A + (1-a^B)\delta \tilde{v}^A]. \tag{19}$$

It is immediate that (18) and (19) correspond term by term to (8) and (9), respectively. Again, focusing on the case in which $s = 1$ and $\alpha = 0$, we know from the analysis of the mixed-strategy equilibrium that if

$$a^A = \frac{\delta(1-p^A) - (1-\delta)(c-1)}{\delta p^A(c-1)},$$
$$a^B = \frac{\delta p^A - (1-\delta)(c-1)}{\delta(1-p^A)(c-1)},$$

then both parties are indifferent between implementing and procrastinating, and hence it is ruling party i's best response that it implements the fraction a^i of the remainder of the policy. It is also be readily verified that V^A and $\tilde{V}^A$ are indeed linear functions of θ as we have guessed. We record this finding in the following proposition.

Proposition 4 *If the cost of the policy is in the intermediate range where the mixed-strategy equilibrium exists, there also co-exists an equilibrium in which the policy is gradually implemented. Each party implements a constant fraction of the remainder of the policy when in office in such a way that the other party is indifferent between implementing and procrstinating when in office.*

5.3 The Effects of Party Predominance

Without loss of generality, we continue to assume that $s = 1$ and $\alpha = 0$ to simplify exposition. As mentioned earlier, equilibria as depicted in the lower panel of Figure 2 will arise when party A is strictly predominant ($p^A > 1/2$). If both parties are perfectly symmetrical (i.e., $p^A = 1/2$), then the threshold implementation costs that demarcate the different equilibria are the same for the two parties: $\delta/2$ and $\delta/(2-\delta)$ as indicated in Figure 3. The analysis is the same as in the asymmetric case except that it is simpler. So, we do not bother to repeat it. Consequently, under symmetry, there are only three equilibrium outcomes corresponding to the different values of $(c-1)/c$: (i) when $(c-1)/c < \delta/2$, both parties implement when in office; (ii) when $\delta/2 < (c-1)/c < \delta/(2-\delta)$ there are multiple equilibria including (a) a mixed strategy equilibrium similar to the asymmetric case, (b) A implements and B procrastinates, and (c) B implements and A procrastinates; (iii) when $\delta/(2-\delta) < (c-1)/c$, both parties procrastinate. As we depart from symmetry and let party A become predominant, i.e. let p^A increases from 1/2, both threshold implementation costs increase for party A while they decrease for party B (so a situation illustrated in the lower panel of Figures 2 arises) so that the range of $(c-1)/c$ that supports multiple equilibria shrinks. If the parties become so asymmetric that $p^A > \bar{p}$, where

$$\bar{p} = \frac{1-\sqrt{1-\delta}}{\delta} \in \left(\frac{1}{2}, 1\right),$$

then δp^A exceeds $\delta(1-p^A)/(1-\delta p^A)$. In this case, the size of the range of $(c-1)/c$ that supports multiple equilibria shrinks to zero, and so the multiple equilibria shown in Figure

2 will disappear.

Party A's predominance increases the chance of policy implementation if the original symmetric equilibrium is characterized by procrastination of the predominated party or both, for example when implementation cost is relatively high so that $(c-1)/c$ is greater than $\delta/(2-\delta)$. In this case, both parties procrastinate when in office if they are symmetrical, but ruling party A will implement the policy (while ruling party B still procrastinates) if it becomes sufficiently predominant. This is because the range of $(c-1)/c$ that supports Equilibrium Type (iv) expands at the expense of that of Equilibrium Type (v) (and Type (iii) for that matter). On the contrary, party A's predominance may hinder policy implementation if the implementation cost is relatively small so that the original symmetric equilibrium is characterized by immediate implementation by whoever is in office. The explanation is the following. When $(c-1)/c$ is smaller than $\delta/2$, the policy is implemented immediately if the two parties are symmetrical. As party A becomes sufficient predominant, however, the policy is implemented only when party A is in office because the range of $(c-1)/c$ that supports Equilibrium Type (ii) expands at the expense of that of Equilibrium Type (i) (and Type (iii) for that matter). Thus, party A's predominance causes a possible implementation delay in this case.

Now refer to the asymmetric case shown in the lower panel of Figure 2. In the equilibrium where ruling party A implements while ruling party B procrastinates [i.e. (ii) and (iii b) and (iv)], the probability that the policy is implemented in each period conditional on the event that the policy has not been implemented (or "hazard rate") equals p^A, which is greater than 1/2. This hazard rate of implementation increases as party A becomes more predominant.[13]

In the mixed-strategy equilibrium (iii a), the hazard rate of implementation in each period equals

$$p^A\sigma^{A*} + (1-p^A)\sigma^{B*} = \frac{\delta - 2(1-\delta)(c-1)}{\delta(c-1)}$$

[13] Party A's predominance, however, lowers the probability of policy implementation in the equilibrium where party A always procrastinates when in office while party B always implements when in office (in equilibrium (iii c)). The hazard rate of implementation in each period is $1-p^A$, which is less than 1/2, and it decreases as party A becomes more predominant. This is admittedly a perverse outcome, but it does not affect the broad picture that the unconditional probability (on c) of procrastination decreases with party predominance.

in each period. In this case, the hazard rate of implementation does not depend on party A's degree of predominance; an increase in p^A leads to an increase in σ^{B*} and a decrease in σ^{A*} so that parties A and B are kept indifferent between implementing and procrastinating when in office. This hazard rate of implementation, however, increases if δ increases or c decreases.[14]

We summarize the above discussion by the following proposition.

Proposition 5 *For a given value of $(c-1)/c$, if the original equilibrium is characterized by procrastination of the predominated party or both parties, an increase in party predominance increases the hazard rate of implementation. But if the original equilibrium is characterized by immediate implementation by both parties, an increase in party predominance can lead to procrastination.*

The first part of the proposition is intuitive, given that the source of procrastination is political uncertainty. As party predominance increases, political uncertainty decreases, which should lead to less procrastination. However, the second part of proposition is counter-intuitive, as less political uncertainty can actually lead to more procrastination.

5.4 Cooperative Equilibrium with Gradual Policy Implementation

We have shown that if the implementation cost is large enough, there exists a subgame perfect equilibrium in which neither party implements the policy. In this case, we have $X_0^i < 0$ (see (17) for the case where $s = 1$), for $i = A, B$, so that each ruling party would obtain negative utility from implementing any positive fraction of the policy. Nevertheless, each ruling party wishes that the policy be implemented sometime in the future since X_t^i is positive if t is large enough. To see this claim, first note that

$$X_t^i = \beta_t^i \left[\sum_{k=0}^{\infty} \frac{\beta_{t+k}^i}{\beta_t^i} - c \right]. \tag{20}$$

As we have seen in Section 3, the behavior of present-biased preferences is very similar to that of geometric discounting far off in the future, i.e., β_{k+1}^i/β_k^i converges to δ as k tends to

[14] Note that this result needs to be modified when $s > 1$.

infinity. Thus, $\beta_{t+k}^i/\beta_t^i = \Pi_{m=0}^{k-1}(\beta_{t+m+1}^i/\beta_{t+m}^i)$ approaches δ^k as t gets larger and larger, and hence the expression in square brackets on the right-hand side of (20) converges to $\sum_{k=0}^{\infty}\delta^k - c$ as t tends to infinity. Since $\sum_{k=0}^{\infty}\delta^k - c > 0$ under the assumption $1/(1-\delta) > c$, we have $X_t^i = \beta_t^i\left[\sum_{k=0}^{\infty}(\beta_{t+k}^i/\beta_t^i) - c\right] > 0$ as t exceeds a certain level. This contrasts sharply with the case of myopia. If the ruling parties are simply myopic (heavily discounting the future with geometric discounting), no ruling party wishes that the policy be implemented in the future if it would obtain negative utility from implementing today. Time inconsistency arises precisely because the parties are present-biased when they are in office.

It follows from $X_0^i < 0$ that if the ruling party expects all future ruling parties to refrain from implementing the policy, it should also stay away from the policy. That is, no ruling party wants to be the last to implement a positive fraction of the policy. The strategy profile in which $a_t = 0$ for any t is a subgame perfect equilibrium as we have seen. This is certainly bad news for the citizens. Although the policy is socially beneficial, there is a possibility of indefinite procrastination. Does there exist any subgame perfect equilibrium in which some future ruling parties at least implement part of the policy?

To answer this question, first note that no ruling party would want to implement the policy to completion since it would bear a utility loss from implementing the last part of it. Therefore, if the policy is indivisible (contrary to our original assumption), then the policy will never be implemented. Thus, we have the following proposition.

Proposition 6 *If the cost of the policy is so high that $X_0^i < 0$, for $i = A, B$, and if the policy is not divisible, then the policy never gets implemented even though it is socially beneficial.*

The proposition says that in case partial implementation of the policy is not feasible, there is indefinite procrastination if the net social benefit is too small.

Indeed, as shown below, if the policy is to be implemented at all, it must be implemented gradually to assure non-negative welfare for every ruling party in the future. Moreover, the policy implementation process must continue indefinitely; otherwise, the ruling party that implements the policy to completion would suffer welfare loss from the part of the policy it implements. The following analysis presents such a gradual implementation equilibrium.

We shall show that when $X_0^i < 0$, a symmetric gradual implementation equilibrium exists if $\sum_{k=0}^{\infty} X_k^i > 0$ for any $i = A, B$, i.e., the simple sum of all current and future utility flows is positive for both parties. The situation in which $\sum_{k=0}^{\infty} X_k^i > 0$ arises if c is relatively small in the high-cost range. The following lemma implies that $X_k^i > 0$ for all $k \geq 1$ when c takes on a value such that $X_0^i = 0$. This in turn implies, by continuity, that even if $X_0^i < 0$, it is possible that $\sum_{k=0}^{\infty} X_k^i > 0$. This happens when c is sufficiently small, as X_k^i decreases in c.

Lemma 5 *If $\alpha < 1$, then $X_t^i > \beta_t^i X_0^i$ for any $t \geq 1$.*

The proof of Lemma 5 is relegated to the Appendix. Under the usual geometric discounting preferences such that $\beta_t^i = \delta^t$, X_t^i would be equal to $\beta_t^i X_0^i$. Under the present-biased preferences, however, the current ruling party puts a disproportionately high weight on the cost incurred in the current period, and so X_0^i is disproportionately small.

Now, consider the stationary action profile, symmetric between the two parties, such that regardless of which party is in office, $a_t = a(1-a)^t$ for some constant $a \in (0,1)$. According to this action profile, both parties implement the fraction a of the remainder of the policy whenever they are in office, and this process continues indefinitely. Consequently, the relevant welfare for the party in office in period t as evaluated in that period equals

$$\sum_{k=0}^{\infty} \left[a(1-a)^k X_k^i \right]. \tag{21}$$

Lemma 6 *Suppose $\sum_{k=0}^{\infty} X_k^i > 0$. Then, there exists $\bar{a} \in (0,1)$ such that for any $a \in (0, \bar{a})$, the relevant welfare for the party in office in period t given by (21) is positive.*

Proof: We first notice that $\sum_{k=0}^{\infty}(1-a)^k X_k^i$ converges to $\sum_{k=0}^{\infty} X_k^i > 0$ as $a \to 0$. Thus, there exists an $\bar{a}$ such that for any $a \in (0, \bar{a})$, $\sum_{k=0}^{\infty}(1-a)^k X_k^i > 0$, and hence $\sum_{k=0}^{\infty} a(1-a)^k X_k^i > 0$.

Q.E.D.

Can this gradual implementation scheme with $a \in (0, \bar{a})$ be supported as a subgame perfect equilibrium? The answer is "yes" as the following strategy profile is subgame perfect.

$$a_t = \begin{cases} a(1-a)^t & \text{if there has been no deviation from } a_k = a(1-a)^k \text{ for all } k \leq t-1 \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

Hence, we obtain the following proposition.

Proposition 7 *If the cost of the policy is sufficiently high that $X_0^i < 0$ for $i = A, B$, but sufficiently small that $\sum_{k=0}^{\infty} X_k^i > 0$ for $i = A, B$, there exists a subgame perfect equilibrium in which every ruling party implements a constant fraction of the remainder of the policy so that the implementation process goes on indefinitely.*

Proof: We show here that the strategy profile (22) is subgame perfect. Since indefinite procrastination is a subgame perfect equilibrium, we need only show that no ruling party has an incentive to deviate from the prescribed actions when there has been no deviation in the past. According to the prescribed action profile, if there has been no deviation, the ruling party in period t is to choose $a_t = a(1-a)^t$, receiving positive welfare from its action (Lemma 6). If it chooses some other level of a_t, on the other hand, the equilibrium path would switch to the "punitive equilibrium" of indefinite procrastination, making the present value of future utility flows zero. Since the utility flow from choosing a positive a_t for the ruling party in period t is negative, the discounted sum of utility flows would be non-positive if it chooses any a_t other than $a(1-a)^t$. Hence, the ruling party in period t is better off conforming to the equilibrium path than choosing any other levels of a_t. Therefore, it will choose $a_t = a(1-a)^t$ if there has been no deviation before period t. **Q.E.D.**

This cooperative equilibrium exists because both parties have preferences characterized by hyperbolic discounting. Based on the analysis in this paper, we can explain a ruling party's possible procrastination in two different ways. First, it may prefer the other party to implement the policy in the near future, rather than implementing it itself. Second, the present value of utility flows from implementing the policy may become greater if the policy is implemented some time in the future due to hyperbolic discounting. The first one was a predominant cause of the mixed-strategy (and non-cooperative gradual implementation) equilibrium derived in subsection 5.1. The second one, on the other hand, is the primary cause of this cooperative gradual implementation equilibrium. A ruling party has an incentive to implement part of the policy only when a significant portion of the policy is sufficiently delayed so that the entire process of policy implementation yields a positive present value of utility flows.

6 Concluding Remarks

We present a theory to explain government procrastination as a consequence of present-bias resulting from the political uncertainty in a two-party political system. Present-bias arises because a party's probability of getting elected in the future is less than one, and because it puts more weight on the flow of net social benefit of the policy when it is in office than when it is not. As a result, the ruling party in a two-party system often procrastinate about implementing socially beneficial policies that carry upfront costs but yield long-term benefits. Another way to look at it is that a ruling party's implementation of the policy today confers positive intertemporal externalities on future rival ruling parties, leading to too little implementation.

We find that there is an array of equilibria, which can be categorized according to the cost-benefit ratio of the policy. The procrastination problem tends to get more serious as the cost-to-benefit ratio gets higher. When the cost is relatively low, there is no procrastination problem. When the cost is intermediate, the parties are in a war of attrition. As the ruling party cannot capture all the future benefits from implementing the policy, there is certain degree of relunctance to implement it immediately or completely. There are various forms of procrastination, such as having a probability of implementation less than one in each period, or gradual implementation by all ruling parties. When the cost is relatively high, the policy may be procrastinated indefinitely, though there may co-exist equilibria in which the policy is implemented gradually in a cooperative manner.

Our theory predicts that a government with a more predominant party tends to procrastinate less. It also predicts that the predominant party procrastinates less than the predominated party. Finally, it predicts that socially desirable policies are more likely to be implemented gradually when the degree to which one party predominates the other is lower. It would be interesting to test these hypotheses.

One can easily derive corresponding results when the government is faced with implementing a policy that confers immediate benefits and demands future flows of costs. In this setting, it is expected that a present-biased government may implement a policy which is not socially beneficial. For example, the ruling party may run too large a budget deficit.

A possible extension of this research is to endogenize the probability of a party being elected. In our model, citizens are far-sighted and wish that the government implement the policy entirely as soon as possible. If this effect dominates, parties would have more incentive to implement the policy if the probability of being re-elected is endogenized; consequently, the equilibrium delay would be shortened. In reality, however, it is equally plausible that at least part of the citizenry is myopic in their voting behavior so that the ruling party's probability of being re-elected is positively correlated with the net flow of social cost resulting from implementing the policy today. In such a case, parties may have less incentive to implement the policy, so that the equilibrium delay may be lengthened. In any event, endogenizing the ruling party's probability of being re-elected would not change the basic message of this paper.

In addition, this model can be easily applied to address specific policy issues, such as trade liberalization, reduction of budget deficit, or social security reform, by adding more structure.

Appendix

Proof of Lemma 5:

To prove Lemma 5, it suffices to show that $\beta^i_{t+k}/\beta^i_t > \beta^i_k$, or $\beta^i_{t+k} > \beta^i_t\beta^i_k$, for any $t \geq 1$ and $k \geq 1$, since $X^i_t = \beta^i_t\left[\sum_{k=0}^{\infty}(\beta^i_{t+k}/\beta^i_t) - c\right]$ and $X^i_0 = \sum_{k=0}^{\infty}\beta^i_k - c$. Indeed, we only show that $\beta^A_{t+k} > \beta^A_t\beta^A_k$ since party B's counterpart is obvious. Recall equation (5) and define

$$
\begin{aligned}
f(\alpha) \;\equiv\;& \alpha + (1-\alpha)\{p^A + (1-p^A)(s-1)^{t+k}\} \\
& -[\alpha + (1-\alpha)\{p^A + (1-p^A)(s-1)^t\}][\alpha + (1-\alpha)\{p^A + (1-p^A)(s-1)^k\}].
\end{aligned}
$$

It is easy to see that $\beta^A_{t+k} > \beta^A_t\beta^A_k$ if and only if $f(\alpha) > 0$.

Now,

$$
\begin{aligned}
f(0) &= p^A + (1-p^A)(s-1)^{t+k} - [p^A + (1-p^A)(s-1)^t][p^A + (1-p^A)(s-1)^k] \\
&= p^A(1-p^A)[1-(s-1)^t][1-(s-1)^k] > 0,
\end{aligned}
$$

since $-1 < s-1 < 1$. In addition, $f(1) = 0$. Moreover, since

$$
f''(\alpha) = -2[1-p^A-(1-p^A)(s-1)^t][1-p^A-(1-p^A)(s-1)^k] < 0,
$$

the function f is a concave function. Thus, we have shown that $f(\alpha) > 0$ for any $\alpha \in [0,1)$.

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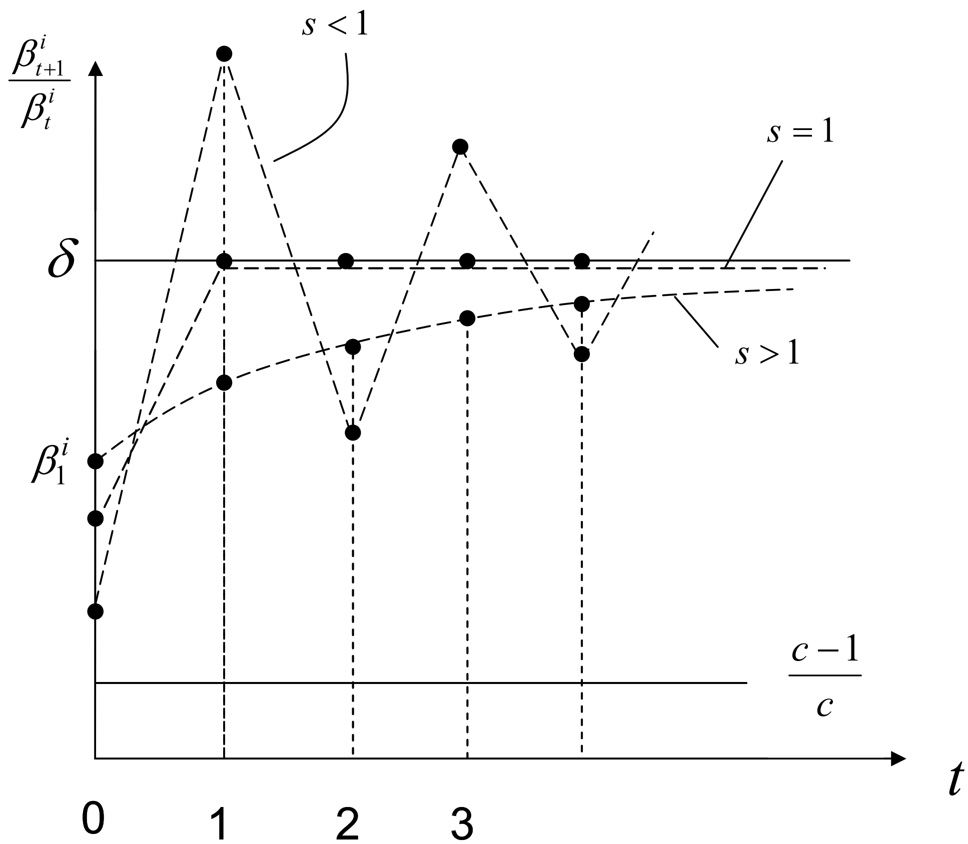

Figure 1. Present-Bias

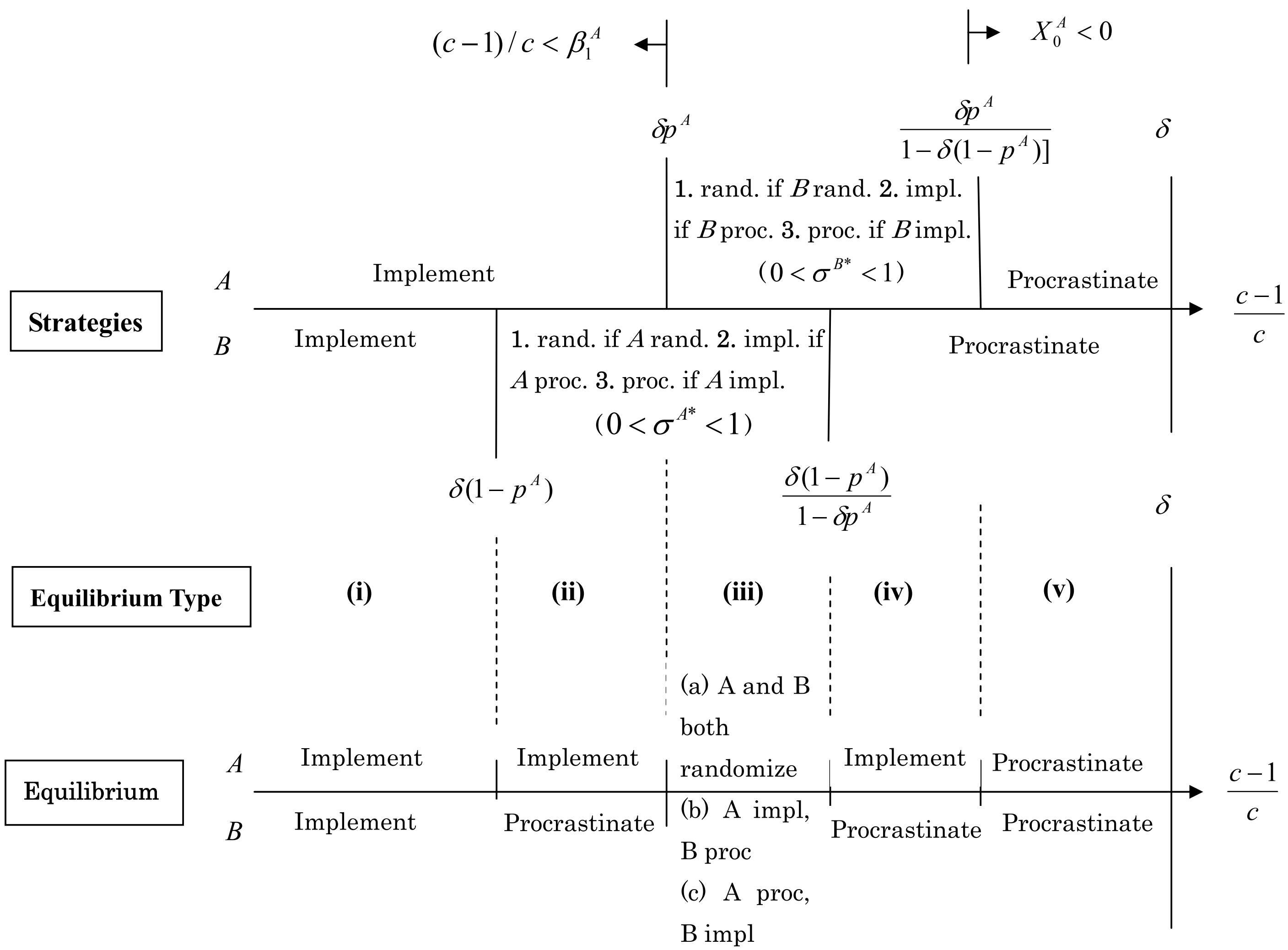

Figure 2. Implementation Strategies and Subgame Perfect Equilibria (case where $\alpha = 0$ and $p^A > 1/2$)

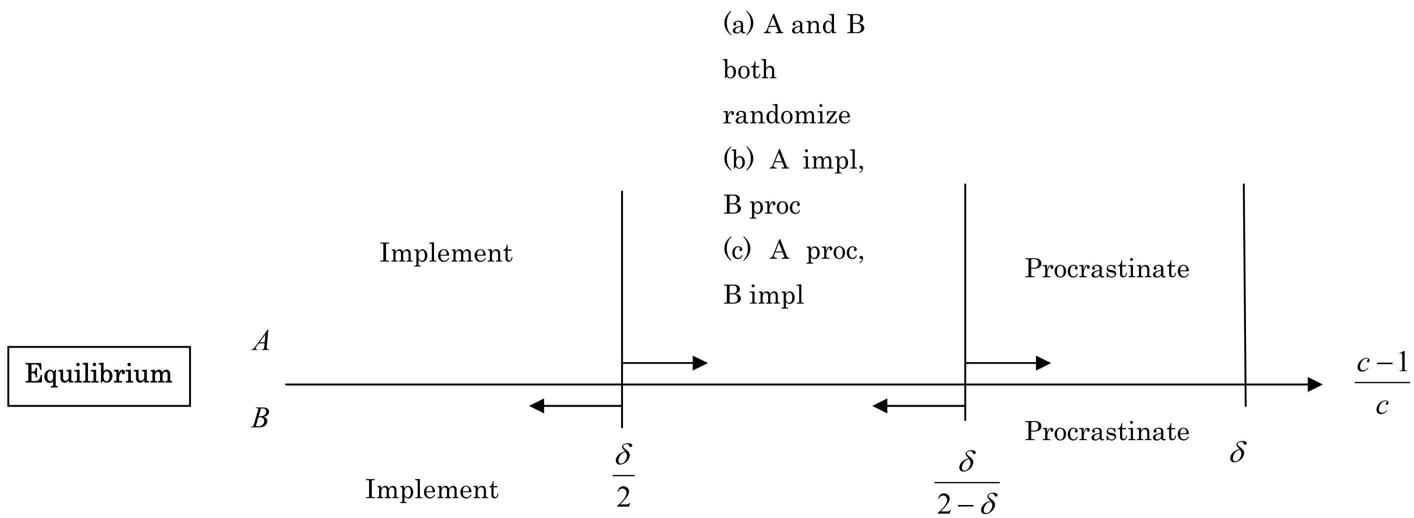

Figure 3. Effect of $p^A \uparrow$ on the critical costs, starting from $p^A = 1/2$. Assume that $\alpha = 0$.