Optimal Redistributive Capital Taxation in a Neoclassical Growth Model
Kevin Lansing
JPubE 1998

1. Judd (1985) and Chamley (1986) obtained results that in the long run, the optimal capital income taxation should be zero. And they claim that the result does not depend on whether government has the ability to borrow or not.

2. Lansing in this paper gave a counterexample under which the optimal capital income tax rate is non-zero in the long run when the government does not have the ability to borrow. The paper shows that government’s ability to borrow matters.

3. The counterexample is similar to Xie (1997) in that “noncontrollability” is involved. The difference is that in Xie, there is a representative agent; in Lansing, there are two classes of individuals: the capitalists and the workers. Capitalists do not work and workers do not save.

4. Capitalists:

   The decision problem faced by capitalists is:

   \[
   \max \int_0^\infty e^{-\rho t} u [c(t)] \, dt, \quad \rho > 0, \tag{1}
   \]

   subject to

   \[
   c(t) + i(t) = \left[1 - \tau_k(t)\right] r(t)k(t) + \tau_k(t)e(t), \tag{2}
   \]

   \[
   \dot{k}(t) = i(t) - \delta k(t), \quad \delta \in [0, 1], \quad k(0) \text{ given}. \tag{3}
   \]

   This will lead to:

   \[
   \dot{c} = \frac{c}{\beta} (\dot{r} - \rho). \tag{6}
   \]

5. Workers:

   Workers neither save nor borrow, but supply one unit of labor inelastically for all t. Their decision problem is trivial: at each point in time they consume all of their income. The workers’ consumption x is given by

   \[
   x = w + TR, \tag{7}
   \]

   where w is the real wage and TR represents lump-sum transfers received from the government.

   The workers’ lifetime utility is given by \( \int_0^\infty e^{-\rho t} v(x) \, dt \), where the function \( v(x) \) is assumed to be increasing and strictly concave in x, with \( \lim_{x \to -\infty} v'(x) = 0 \).

6. The firms:

   \[
   \tau = f'(k), \tag{8}
   \]

   \[
   \omega = f(k) - f'(k) k. \tag{9}
   \]

7. When the government has no ability to borrow:

   \[
   TR = \tau_k (r - \delta) k. \tag{10}
   \]
9. The government's problem is to:

\[
\max_{\dot{r}, k, c} \int_0^\infty e^{-\rho t} \{ \gamma v (f(k) - \delta k - \dot{r}k) + u(c) \} \, dt,
\]

subject to

\[
\dot{c} = \frac{c}{\beta} (\dot{r} - \rho), \tag{12}
\]

\[
\dot{k} = \dot{r}k - c, \quad k(0) \text{ given}, \tag{13}
\]

\[
\dot{r} \geq 0, \tag{14}
\]

\[
\lim_{t \to \infty} e^{-\rho t} u'(c)k = 0. \tag{15}
\]

10. If we follow Judd (1985):

The current-value Hamiltonian for the government's problem is

\[
H_g = \gamma v (f(k) - \delta k - \dot{r}k) + u(c) + q_1 (\dot{r}k - c) + q_2 \frac{c}{\beta} (\dot{r} - \rho) + \eta \dot{r}, \tag{16}
\]

where \( q_1 \) and \( q_2 \) are the multipliers associated with (12) and (13) and \( \eta \) is the Kuhn-Tucker multiplier on the inequality constraint (14). The first-order conditions for this problem are

\[
\frac{\partial H_g}{\partial k} = \gamma v'(x)(f'(k) - \delta - \dot{r}) + q_1\dot{r} = \rho q_1 - q_1, \tag{17a}
\]

\[
\frac{\partial H_g}{\partial c} = u'(c) - q_1 + q_2 \frac{1}{\beta} (\dot{r} - \rho) = \rho q_2 - \dot{q}_2, \tag{17b}
\]

\[
\frac{\partial H_g}{\partial \dot{r}} = -\gamma v'(x)k + q_1k + q_2 \frac{c}{\beta} + \eta = 0, \tag{17c}
\]

\[
\frac{\partial H_g}{\partial q_1} = \dot{r}k - c = \dot{k}, \tag{17d}
\]

\[
\frac{\partial H_g}{\partial q_2} = \frac{c}{\beta} (\dot{r} - \rho) = \dot{c}. \tag{17e}
\]

11. It is straightforward to show that the capital income taxation has to be zero at the steady state.

12. However, (17a) and (17b) are valid only if the two variables \( k \) and \( c \) are independent from each other. But it is easy to see from (12), (13), and (15) that when the utility function is logarithmic, consumption of the capitalist and the capital stock are closely linked:

\[ c = \rho k. \]

13. Given this close relationship, the government's problem can be reformulated (as in Xie 1997). And Lansing has the following result:

**Proposition 2.** When there is no market for government bonds and \( u(c) = \log(c) \), the sign of the steady-state optimal tax on capital income is governed by the following conditions:

\[
(i) \quad \tau_k(\infty) > 0 \quad \text{if} \quad \rho > \frac{1}{\gamma v'(x)k'},
\]

\[
(ii) \quad \tau_k(\infty) = 0 \quad \text{if} \quad \rho = \frac{1}{\gamma v'(x)k'},
\]

\[
(iii) \quad \tau_k(\infty) < 0 \quad \text{if} \quad \rho < \frac{1}{\gamma v'(x)k'}.
\]
14. Under the assumption of \( f(k) = k^\theta \) and \( v(x) = \ln x \), Lansing obtains a closed-form solution and the following comparative statics analysis:

\[
\frac{\partial \pi(\infty)}{\partial \rho} = \frac{\delta (1 - \theta) (1 + \gamma \theta)}{[\rho \theta (1 + 2\gamma) - \delta (1 - \theta)]^2} > 0,
\]

\[
\frac{\partial \pi(\infty)}{\partial \gamma} = \frac{\rho \theta [\rho (2 - \theta) + \delta (1 - \theta)]}{[\rho \theta (1 + 2\gamma) - \delta (1 - \theta)]^2} > 0,
\]

\[
\frac{\partial \pi(\infty)}{\partial \theta} = \frac{\rho [\rho (1 + 2\gamma) + \delta (1 + \gamma)]}{[\rho \theta (1 + 2\gamma) - \delta (1 - \theta)]^2} > 0,
\]

\[
\frac{\partial \pi(\infty)}{\partial \delta} = \frac{-\rho (1 - \theta) (1 + \gamma \theta)}{[\rho \theta (1 + 2\gamma) - \delta (1 - \theta)]^2} < 0.
\]

An increase in the rate of time preference \( \rho \) reduces the perceived benefits of saving relative to consumption. This motivates the government to raise the tax rate. As \( \gamma \) goes up, the government increasingly favors workers and undertakes more redistribution. The parameter \( \theta \) represents capital’s share of total output. As \( \theta \) increases, before-tax income inequality rises and the government chooses a higher tax rate. Finally, a higher depreciation rate \( \delta \) means that capitalists need to save more in order to maintain the capital stock. A lower tax rate provides the incentive to do so.

15. Lansing went on to prove that when the government is allowed to borrow, or when the government has the option of consumption tax, the close link between consumption and capital stock is broken and Judd’s result obtains.

16. Lansing also looked at the Chamley setup (hence more similar to Xie, section 6, 1997). He showed that when government has no ability to borrow, indeterminacy may arise, hence capital income taxes in the long run are indeterminate. Xie allowed the existence of government bonds and showed that in the long run, capital income taxation has to be zero in Chamley setup.